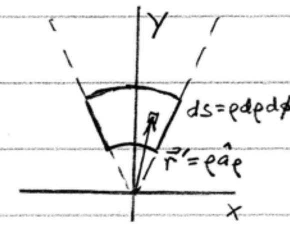
1. (a.) $Q = \iint \rho_s \, ds = \rho_s \iint ds = 5\times10^{-9} \int_{\pi/3}^{2\pi/3}\int_{0.1}^{0.2} \rho \, d\rho \, d\phi = 5\times10^{-9}\left[\frac{\rho^2}{2}\right]_{0.1}^{0.2}[\phi]_{\pi/3}^{2\pi/3} = \boxed{78.5\,pC}$

(b.) $V = \frac{1}{4\pi\epsilon_0}\iint \frac{\rho_s}{R} ds$, $\vec{R} = \vec{r} - \vec{r}\,'$, $\vec{r} = \vec{0}$, $\vec{r}\,' = \rho\hat{a}_\rho$, $\vec{R} = -\rho\hat{a}_\rho$, $R = |\vec{R}| = \rho$,

$ds = \rho\, d\rho\, d\phi$, $V = \frac{\rho_s}{4\pi\epsilon_0}\int_{\pi/3}^{2\pi/3}\int_{0.1}^{0.2} \frac{\rho\, d\rho\, d\phi}{\rho} = \frac{\rho_s}{4\pi\epsilon_0}[\rho]_{0.1}^{0.2}[\phi]_{\pi/3}^{2\pi/3} = \boxed{4.71\ V}$



(c.) $\vec{E} = \frac{1}{4\pi\epsilon_0}\iint \frac{\rho_s}{R^2}\hat{a}_R\, ds$, $\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = -\hat{a}_\rho = -\cos\phi\,\hat{a}_x - \sin\phi\,\hat{a}_y$

$\vec{E} = \frac{\rho_s}{4\pi\epsilon_0}\int_{\pi/3}^{2\pi/3}\int_{0.1}^{0.2} \frac{[-\cos\phi\,\hat{a}_x - \sin\phi\,\hat{a}_y]}{\rho^2}\rho\, d\rho\, d\phi = \frac{\rho_s}{4\pi\epsilon_0}\left[-\hat{a}_x\int_{\pi/3}^{2\pi/3}\int_{0.1}^{0.2}\frac{\cos\phi}{\rho}d\rho\, d\phi - \hat{a}_y\int_{\pi/3}^{2\pi/3}\int_{0.1}^{0.2}\frac{\sin\phi}{\rho}d\rho\, d\phi\right]$

$= \frac{\rho_s}{4\pi\epsilon_0}\left\{-\hat{a}_x[\sin\phi]_{\pi/3}^{2\pi/3}[\ln\rho]_{0.1}^{0.2} - \hat{a}_y[-\cos\phi]_{\pi/3}^{2\pi/3}[\ln\rho]_{0.1}^{0.2}\right\}$

$= \frac{-\rho_s}{4\pi\epsilon_0}\hat{a}_y(1)(\ln 2) = \boxed{-31.1\,\hat{a}_y\ V/m}$

2. (a.) $\vec{D} = \epsilon_0\vec{E} = \epsilon_0\left[2xyz^3\hat{a}_x - (1-x^2)z^3\hat{a}_y - 3(1-x^2)yz^2\hat{a}_z\right]$, $\vec{D}(2,3,1) = \boxed{\epsilon_0\left[12\hat{a}_x + 3\hat{a}_y + 27\hat{a}_z\right] C/m^2}$

(b.) $\rho_v = \nabla\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \epsilon_0\left[2yz^3 - 6(1-x^2)yz\right]$, $\rho_v(2,3,1) = 60\epsilon_0 = \boxed{531\,pC/m^3}$

(c.) $\hat{a}_F = \frac{\vec{F}}{|\vec{F}|} = \frac{3\hat{a}_x + 4\hat{a}_y}{\sqrt{3^2+4^2}}$, $(\vec{E}\cdot\hat{a}_F)\hat{a}_F = \left[(12\hat{a}_x + 3\hat{a}_y + 27\hat{a}_z)\cdot\frac{1}{5}(3\hat{a}_x + 4\hat{a}_y)\right]\frac{1}{5}(3\hat{a}_x + 4\hat{a}_y) =$

$= (36+12)\frac{1}{25}(3\hat{a}_x + 4\hat{a}_y) = \boxed{\frac{48}{25}(3\hat{a}_x + 4\hat{a}_y)\ V/m}$

(d.) $V_{AB} = -\int_A^B \vec{E}\cdot d\vec{l}$, $d\vec{l} = (-dx)(-\hat{a}_x) = dx\,\hat{a}_x$, $\vec{E}(y=3, z=1) = 6x\hat{a}_x - (1-x^2)\hat{a}_y - 9(1-x^2)\hat{a}_z$

$\vec{E}\cdot d\vec{l} = 6x\,dx$, $V_{AB} = -\int_2^0 6x\,dx = -6\int_{2}^{0} x\,dx = -6\left[\frac{x^2}{2}\right]_2^0 = (-6)(-2) = \boxed{12V}$

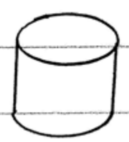
3. (a.) $\vec{E} = -\nabla V = -\frac{\partial V}{\partial\rho}\hat{a}_\rho - \frac{1}{\rho}\frac{\partial V}{\partial\phi}\hat{a}_\phi = \boxed{-5\cos^2\phi\,\hat{a}_\rho + 10\sin\phi\cos\phi\,\hat{a}_\phi\ (V/m)}$

(b.) $\vec{F} = Q\vec{E}$, $\vec{E}(40^\circ, 1) = -5\hat{a}_\rho$, $\vec{F} = (2\times10^{-6})(-5\hat{a}_\rho) = \boxed{-10^{-5}\hat{a}_\rho\ N}$

(c.) $W_{AB} = QV_{AB} = -Q\int_A^B \vec{E}\cdot d\vec{l}$, $\vec{E}(\phi=0^\circ, z=1) = -5\hat{a}_\rho\ (V/m)$, $d\vec{l} = d\rho\,\hat{a}_\rho$

$W_{AB} = -2\times10^{-6}\int_4^8(-5\hat{a}_\rho)\cdot(d\rho\,\hat{a}_\rho) = 10^{-5}[\rho]_4^8 = 4\times10^{-5}\ J = \boxed{40\,\mu J}$

(d.)



$Q_{ENC} = \oint_S \vec{D}\cdot d\vec{s}$, $\vec{D} = \epsilon_0\vec{E} = \epsilon_0\left[-5\cos\phi\,\hat{a}_\rho + 10\sin\phi\cos\phi\,\hat{a}_\phi\right]$

S_1 - CYLINDRICAL SURFACE ($\rho=1$), $d\vec{s}_1 = \rho\, d\phi\, dz\,\hat{a}_\rho$, $\vec{D}_1\cdot d\vec{s}_1 = -5\epsilon_0\cos\phi\, d\phi\, dz$

S_2 - TOP ENDCAP, $d\vec{s}_2 = \rho\, d\rho\, d\phi\,\hat{a}_z$, $\vec{D}_2\cdot d\vec{s}_2 = 0$

S_3 - BOTTOM ENDCAP, $d\vec{s}_3 = \rho\, d\rho\, d\phi\,(-\hat{a}_z)$, $\vec{D}_3\cdot d\vec{s}_3 = 0$

$\oint_S \vec{D}\cdot d\vec{s} = -5\epsilon_0\int_0^1\int_0^{2\pi}\cos^2\phi\, d\phi\, dz = -5\epsilon_0\left[\frac{\phi}{2} + \frac{1}{4}\sin 2\phi\right]_0^{2\pi}[z]_0^1 = -5\pi\epsilon_0 = \boxed{-139\,\mu C}$