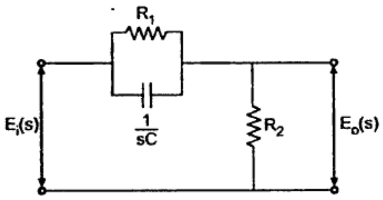
## ***Ans Q#1:***

**Solution :** Take Laplace transform of the network, as shown in the Fig




The parallel combination of $R_1$ and $\frac{1}{sC}$ gives impedance of,

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{R_1 \times \frac{1}{sC}}{R_1 + \frac{1}{sC}} = \frac{R_1}{1 + s R_1 C}$$

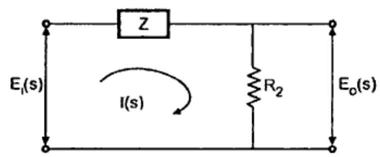



**Fig. 3.17**

Applying KVL to the circuit,

$$E_i(s) = Z\, I(s) + I(s)\, R_2 \qquad \text{... (1)}$$

$$E_o(s) = I(s)\, R_2 \qquad \text{... (2)}$$

$$\therefore \quad I(s) = \frac{E_o(s)}{R_2} \qquad \text{from (2)}$$

Substituting in (1) we get,

$$E_i(s) = I(s)\,[Z + R_2] = \frac{E_o(s)}{R_2}[Z + R_2]$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{R_2}{Z + R_2}$$

Substituting Z, T. F. $= \dfrac{R_2}{\dfrac{R_1}{1 + s R_1 C} + R_2} = \dfrac{R_2 (1 + s R_1 C)}{R_1 + R_2 (1 + s R_1 C)}$

$$= \frac{s R_1 R_2 C + R_2}{R_1 + s R_1 R_2 C + R_2} = \frac{s + \alpha}{s + \beta}$$

where $\alpha = \dfrac{1}{R_1 C}$, $\beta = \dfrac{(R_1 + R_2)}{R_1 R_2 C}$

This circuit is also called **lead compensator**.

## ***Ans Q#2:***

**Solution :** Two forward paths, K = 2,

$$T_1 = G_1 G_3 G_4 G_5 G_6$$

$$T_2 = G_1 G_2 G_6$$

Loops are,

$$L_1 = -G_4 H_1$$

$$L_2 = -G_3 G_4 G_5 H_2$$

$$L_3 = -G_2 H_2$$

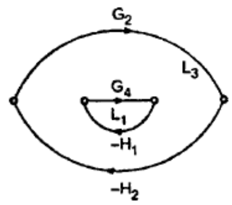



**Fig. 6.14(a) Non touching loops**

Out of these, $L_1$ and $L_3$ is combination of 2 non touching loops

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_3]$$

$\Delta_1$ = Eliminate $L_1$, $L_2$, $L_3$ as all are touching to $T_1$ from $\Delta$

$\therefore \quad \Delta_1 = 1$

$\Delta_2$ = Eliminate $L_2$ and $L_3$, as they are touching to $T_2$, from $\Delta$. But $L_1$ is non touching hence keep it as it is in $\Delta$.

$\therefore \quad \Delta_2 = 1 - [L_1]$

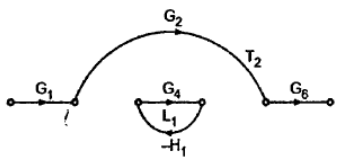



**Fig. 6.14(b) $L_1$ Non touching to $T_2$**

Substitute in Mason's gain formula,

$$\text{T.F.} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$$

$$\boxed{\text{T.F.} = \frac{G_1 G_3 G_4 G_5 G_6 [1] + G_1 G_2 G_6 [1 + G_4 H_1]}{1 + G_4 H_1 + G_3 G_4 G_5 H_2 + G_2 H_2 + G_2 G_4 H_1 H_2}}$$

## *Ans Q#3:*

**Solution :** Arrange the given transfer function as,

$$\frac{Q(s)}{I(s)} = \frac{1}{J\left[s^2+\frac{f}{J}s+\frac{K}{J}\right]} = \frac{\left(\frac{1}{J}\right)}{s^2+\frac{f}{J}s+\frac{K}{J}}$$

Comparing denominator with $s^2+2\xi\omega_n s+\omega_n^2$,

$$\omega_n^2 = \frac{K}{J} \quad \text{i.e.} \quad \omega_n = \sqrt{\frac{K}{J}} \qquad \text{... (1)}$$

and

$$2\xi\omega_n = \frac{f}{J} \quad \text{i.e.} \quad \xi = \frac{f}{2\sqrt{KJ}} \qquad \text{... (2)}$$

Now $M_p = 6\%$ i.e., 0.06

$$\therefore \quad 0.06 = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\therefore \quad \ln(0.06) = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

Solving for $\xi$, $\xi = 0.667$ ... (3)

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 1 \text{ sec}$$

$$\therefore \quad \omega_n = \frac{\pi}{\sqrt{1-(0.667)^2}} = 4.2165 \text{ rad/sec} \qquad \text{... (4)}$$

The Laplace transform of output is Q(s).

Now input is step of 10 Nm hence corresponding Laplace transform is,

$$I(s) = \frac{10}{s}$$

$$\therefore \quad \frac{Q(s)}{\left(\frac{10}{s}\right)} = \frac{1}{Js^2+fs+K}$$

$$\therefore \quad Q(s) = \frac{10}{s(Js^2+fs+K)}$$

The steady state of output can be obtained by final value theorem.

Steady state output $= \lim_{s\to 0} sQ(s)$

$$\therefore \quad 0.5 = \lim_{s\to 0}\frac{s\cdot 10}{s(Js^2+fs+K)} = \frac{10}{K}$$

$\therefore \qquad \mathbf{K = 20}$

Equating (1) and (4), $4.2165 = \sqrt{\frac{K}{J}}$

$\therefore \qquad 4.2165 = \sqrt{\frac{20}{J}}$

$\therefore \qquad \mathbf{J = 1.1249}$

From equation (2), $0.667 = \frac{f}{2\sqrt{KJ}}$

$\therefore \qquad 0.667 = \frac{f}{2\sqrt{20 \times 1.1249}}$

$\therefore \qquad \mathbf{f = 6.3274}$