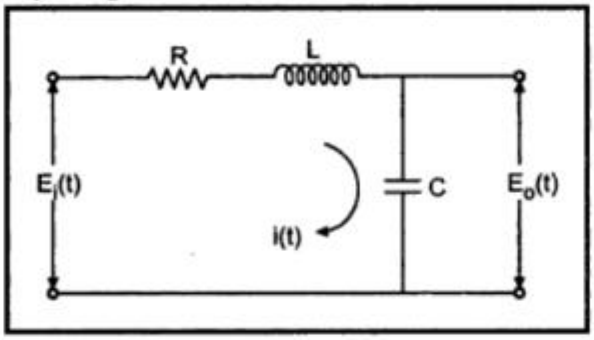
Ex.1. *Find out the T.F. of the given network.*



Sol. : Applying KVL we get the equations as,

$$E_i = iR + L\frac{di}{dt} + \frac{1}{C}\int idt \quad \text{... (1)}$$

$$i/p = E_i \quad ; \quad o/p = E_o$$

Laplace transform of $\int F(t)\,dt = \frac{F(s)}{s}$, ... neglecting initial conditions

and laplace transform of $\frac{df(t)}{dt} = sF(s)$ ... neglecting initial conditions

Take Laplace transform,

$$\therefore \quad E_i(s) = I(s)\left[R + sL + \frac{1}{sC}\right]$$

$$\frac{I(s)}{E_i(s)} = \frac{1}{\left[R + sL + \frac{1}{sC}\right]} \quad \text{... (2)}$$

Now $$E_o = \frac{1}{C}\int idt \quad \text{... (3)}$$

$$\therefore \quad E_o(s) = \frac{1}{sC} I(s)$$

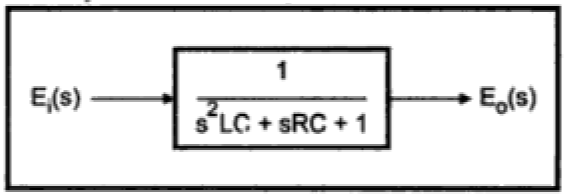
$$\therefore \quad I(s) = sC\, E_o(s) \quad \text{... (4)}$$

Substituting value of I(s) in equation (2)

$$\therefore \quad \frac{sCE_o(s)}{E_i(s)} = \frac{1}{\left[R + sL + \frac{1}{sC}\right]}$$

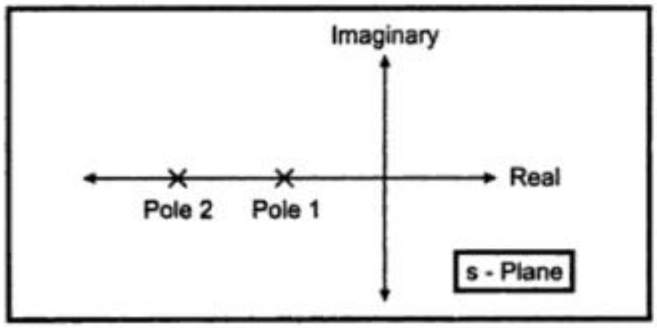
$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{1}{sC\left[R + sL + \frac{1}{sC}\right]} = \frac{1}{RsC + s^2 LC + 1}$$

So we can represent the system as



The characteristic equation is,

$$s^2 + s\frac{R}{L} + \frac{1}{LC} = 0$$

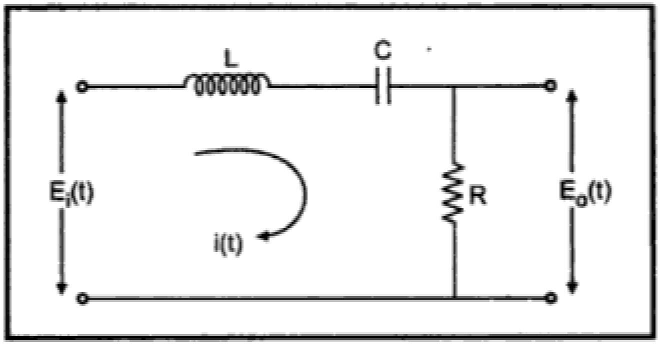


So system is 2nd order and the two poles are, $-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$

T.F. has no zeros.

Now if values of R, L and C selected are such that both poles are real, unequal and negative the corresponding pole-zero plot can be shown as in Fig.

Ex.2. *Find out the T.F. of the given network*



Sol. : Applying KVL we can write,

$$E_i(t) = L\frac{d_i(t)}{dt} + \frac{1}{C}\int i(t)\,dt + i(t)R \qquad \text{... (1)}$$

While $\quad E_o(t) = i(t)R \qquad$ ... (2)

Where $\quad E_i(t) =$ input and $E_o(t) =$ output

Taking Laplace of equations (1) and (2), neglecting the initial conditions.

$$E_i(s) = sLI(s) + \frac{1}{C}\frac{I(s)}{s} + RI(s) \qquad \text{... (3)}$$

$$E_o(s) = I(s)R \qquad \text{... (4)}$$

$$\therefore \quad E_i(s) = I(s)\left[sL + \frac{1}{sC} + R\right] \text{ from (3)}$$

Substituting $\quad I(s) = \frac{E_o(s)}{R}$ from (4) in the above equation we get,

$$E_i(s) = \frac{E_o(s)}{R}\left[sL + \frac{1}{sC} + R\right]$$

$$\therefore \quad E_i(s) = \frac{E_o(s)}{R} \times \left[\frac{s^2LC + 1 + sCR}{sC}\right]$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{sRC}{s^2\,LC + sRC + 1}$$

Ex.3. *The Laplace inverse of the transfer function in time domain of a certain system is e^{-5t} while its input is $r(t) = 2$. Determine its output $c(t)$.*

Sol. : Let T(s) be the transfer function

$$L^{-1}[T(s)] = T(t) = e^{-5t} \quad \text{given}$$

$$r(t) = 2$$

But $$c(t) \neq r(t) \times T(t),$$

it is mentioned earlier that $\frac{c(t)}{r(t)} = T(t)$ is not at all valid in time domain, so

$$c(t) \neq 2e^{-5t}$$

Hence the equation valid according to the definition of transfer function is,

$$T(s) = \frac{C(s)}{R(s)}$$

so $$T(s) = L\{T(t)\} = L\left\{e^{-5t}\right\}$$

$$= \frac{1}{s+5}$$

$$R(s) = \frac{2}{s}$$

$\therefore$ $$\frac{1}{s+5} = \frac{C(s)}{\left(\frac{2}{s}\right)}$$

$\therefore$ $$C(s) = \frac{2}{s(s+5)} = \frac{a_1}{s} + \frac{a_2}{s+5}$$

$\therefore$ $$C(s) = \frac{0.4}{s} - \frac{0.4}{s+5}$$

Taking Laplace inverse of this equation

$$c(t) = 0.4 - 0.4\, e^{-5t}$$

This is the required output expression.

Ex.4. *The transfer function of a system is given by*

$$T(s) = \frac{K(s+6)}{s(s+2)(s+5)(s^2+7s+12)}$$

Determine i) Poles ii) Zeros iii) Characteristic equation and iv) Pole-zero plot in s-plane.

Sol. :

i) **Poles** are the roots of the equation obtained by equating denominator to zero **i.e. roots of**

$$s(s+2)(s+5)(s^2+7s+12) = 0$$

i.e. $s(s + 2)(s + 5)(s + 3)(s + 4) = 0$

So there are 5 poles located at

$s = 0, -2, -5, -3$ and -4

ii) Zeros are the roots of the equation obtained by equating numerator to zero i.e. roots of

$$K(s + 6) = 0$$

i.e. $s = -6$

There is only one zero.

iii) Characteristic equation is one, whose roots are the poles of the transfer function. So it is

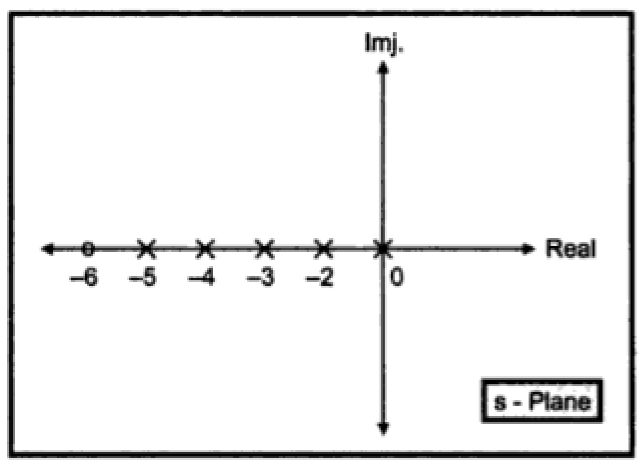
$$s(s + 2)(s + 5)(s^2+7s+12) = 0$$

i.e. $s(s^2+7s+10)(s^2+7s+12) = 0$

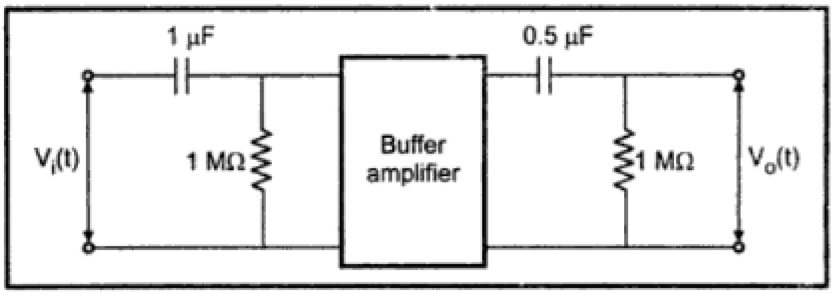
i.e $s^5+14s^4+71s^3+154s^2+120s = 0$

iv) Pole-zero plot

This is shown in the Fig.

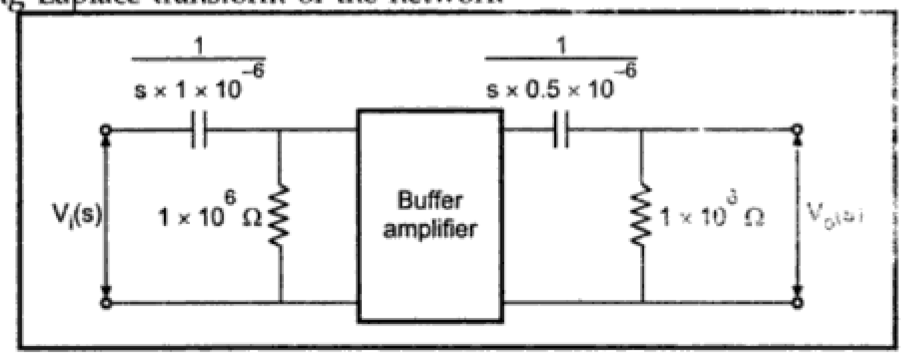


Ex.5. *Find $V_o(s) / V_i(s)$*



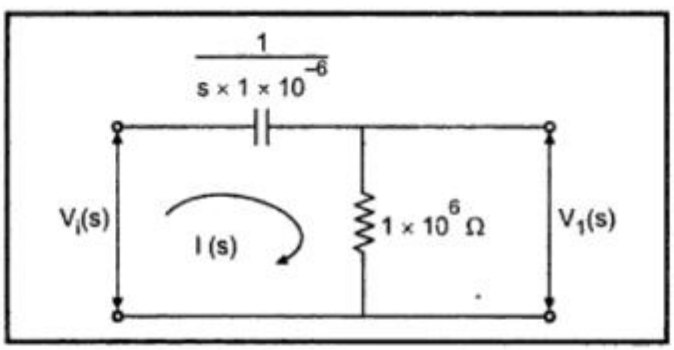
Assume gain of buffer amplifier as 1.

Sol. : Taking Laplace transform of the network



Let us divide the network into two parts

Part 1)



Applying KVL

$$V_i(s) = \frac{1}{s \times 1 \times 10^{-6}} I(s) + 1 \times 10^6 I(s) \quad \text{... (1)}$$

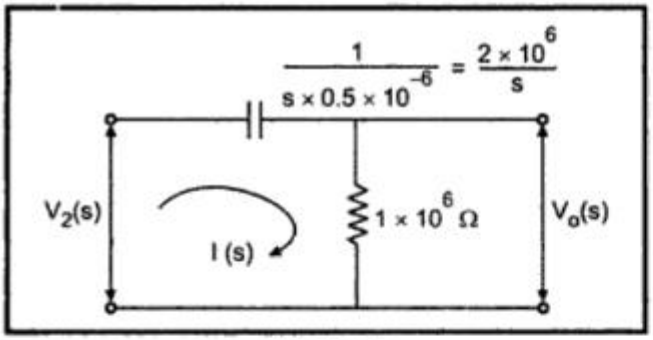
$$V_1(s) = 1 \times 10^6 I(s) \quad \text{... (2)}$$

$$\therefore \quad I(s) = \frac{V_1(s)}{1 \times 10^6}$$

Substituting in (1) $V_i(s) = \left[\frac{10^6}{s} + 10^6\right] I(s) = \left[\frac{10^6 + s\,10^6}{s}\right]\left[\frac{V_1(s)}{10^6}\right]$

$$\therefore \quad \frac{V_1(s)}{V_i(s)} = \frac{s}{s+1}$$

Part 2)



$$\therefore \quad V_2(s) = I(s)\left[\frac{2 \times 10^6}{s} + 1 \times 10^6\right] \quad \text{... (1)}$$

$$V_o(s) = I(s)\,1 \times 10^6 \quad \text{... (2)}$$

$$\therefore \quad I(s) = \frac{V_o(s)}{10^6}$$

Substituting in (1) $V_2(s) = \frac{V_o(s)}{10^6}\left[\frac{2+s}{s}\right]10^6$

$$\therefore \quad \frac{V_o(s)}{V_2(s)} = \frac{s}{s+2}$$

Now gain of buffer amplifier is 1 (unity)

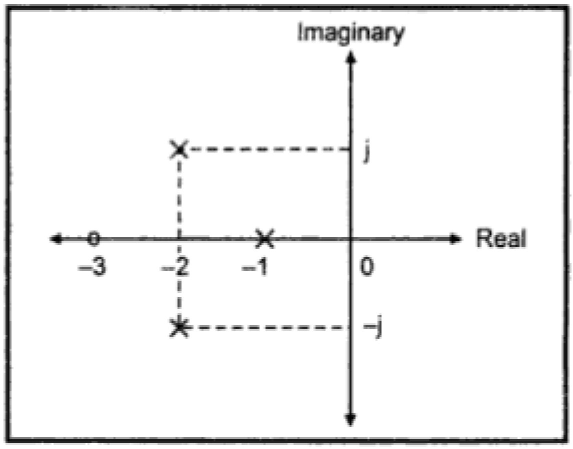
$$\therefore \quad V_1(s) = V_2(s)$$

$$\therefore \quad \left(\frac{s}{s+1}\right)V_i(s) = \frac{(s+2)}{s}V_o(s)$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{s^2}{(s+1)(s+2)}$$

This is the required transfer function.

Ex.6. *Determine the transfer function if the d.c. gain is equal to 10 for the system whose pole-zero plot is shown below.*



Sol. : From pole-zero plot given the transfer function has 3 poles at s = 0, s = –1, –2+j and –2–j. And it has one zero at s = –3.

$$\therefore \quad T(s) = \frac{K(s+3)}{(s+1)(s+2+j)(s+2-j)}$$

$$= \frac{K(s+3)}{(s+1)\left[(s+2)^2-(j)^2\right]}$$

$$= \frac{K(s+3)}{(s+1)\left[s^2+4s+5\right]}$$

Now d.c. gain is value of T(s) at s = 0 which is given as 10.

$$\therefore \quad \text{d.c. gain} = T(s)|_{\text{at } s=0}$$

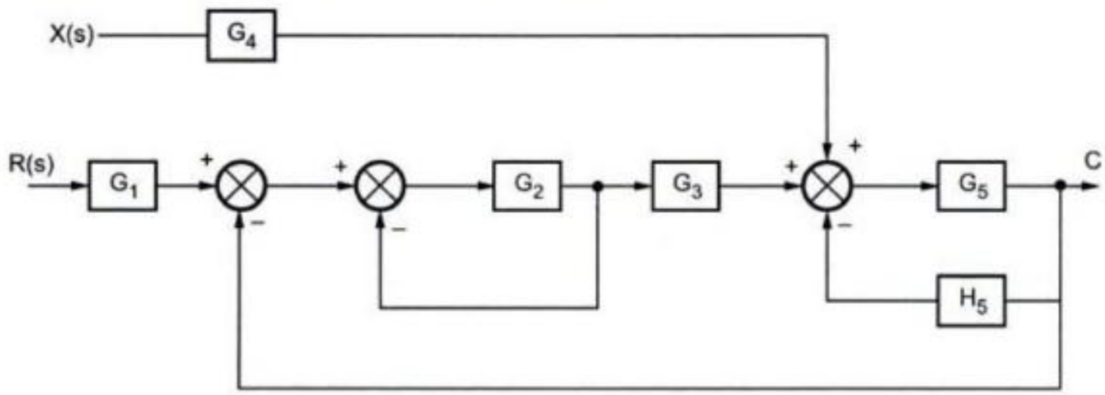
$$\therefore \quad 10 = \frac{K \times 3}{1 \times 5}$$

$$\therefore \quad K = \frac{50}{3} = 16.667$$

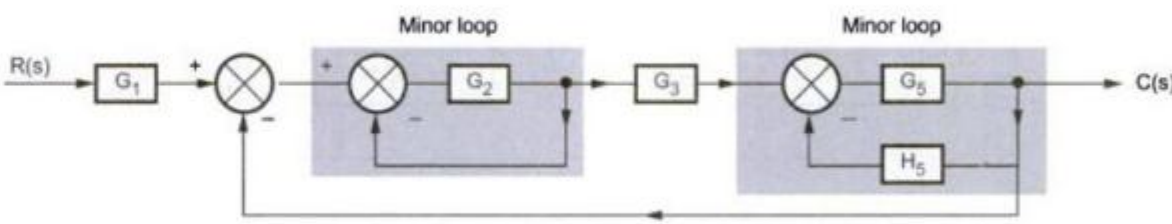
$$\therefore \quad T(s) = \frac{16.667\,(s+3)}{(s+1)(s^2+4s+5)}$$

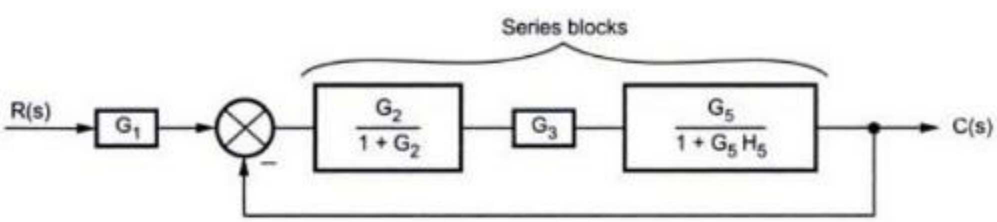
This is the required transfer function.

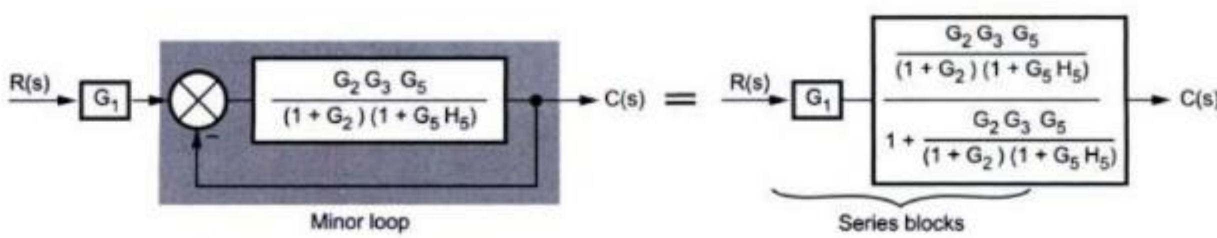
Ex.7. *Using block diagram reduction technique find the transfer function from each input to the output C for the system shown in the Fig.*



Solution : With X(s) = 0, block diagram reduces as,

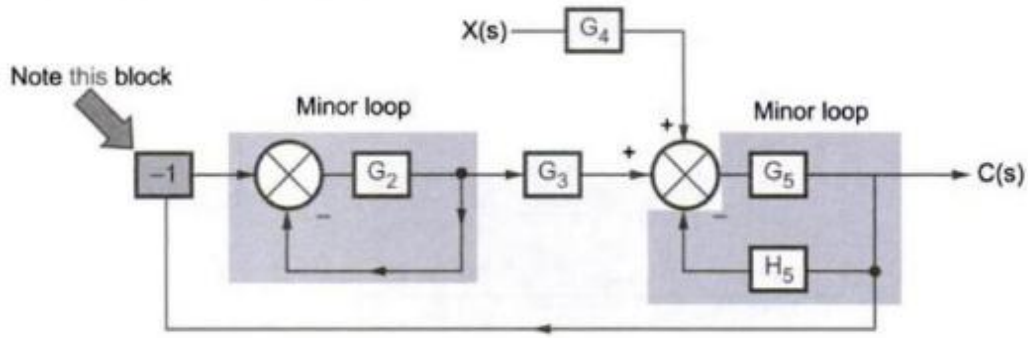


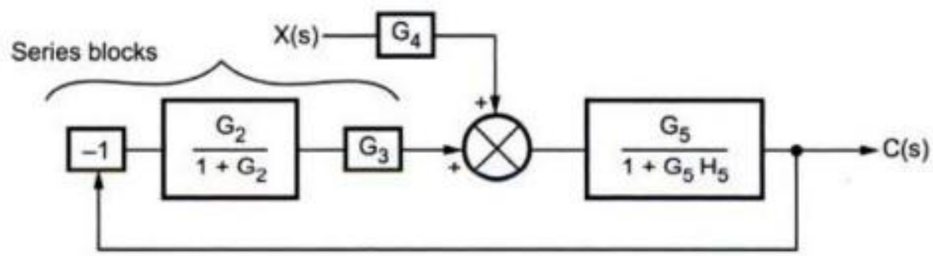




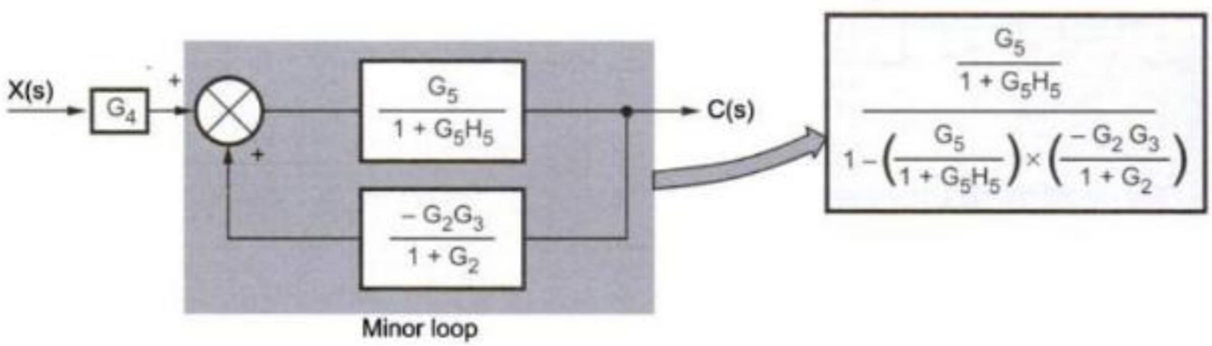
$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_5}{1+G_2+G_5H_5+G_2G_5H_5+G_2G_3G_5}$$

With R(s) = 0, G_1 vanishes but minus sign at summing point must be considered by introducing block of – 1, as shown.



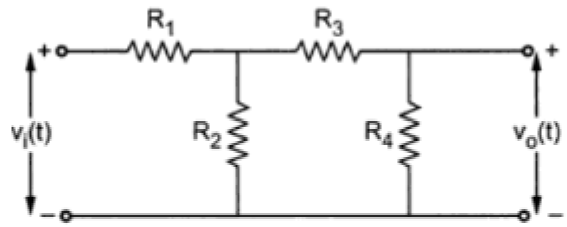


Rearranging the input - output we get,

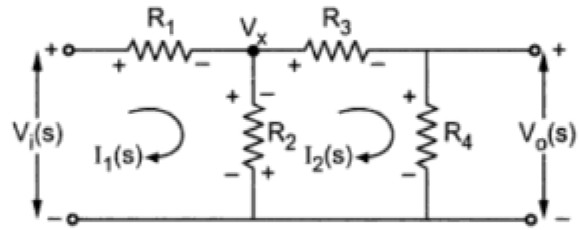


$$\therefore \qquad \frac{C(s)}{X(s)} = \frac{G_4G_5(1+G_2)}{1+G_5H_5+G_2+G_2G_5H_5+G_2G_3G_5}$$

Ex.8. *Obtain the block diagram for the given electrical network.*



Solution : Convert the given network into Laplace domain and assume the currents as shown in the Fig.



The KVL equations for the two loops are,

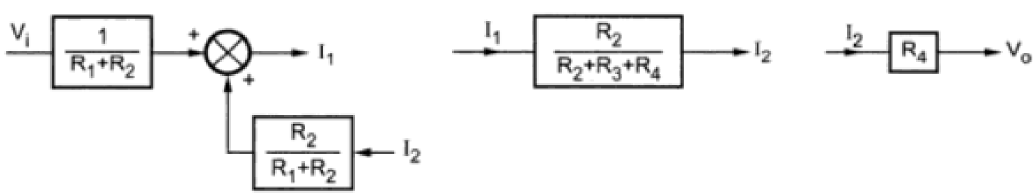
$$-I_1R_1 - I_1R_2 + I_2R_2 + V_i = 0 \qquad \text{i.e} \quad I_1 = V_i\left(\frac{1}{R_1+R_2}\right) + I_2\left(\frac{R_2}{R_1+R_2}\right) \quad \dots (1)$$

$$-I_2R_3 - I_2R_4 - I_2R_2 + I_1R_2 = 0 \qquad \text{i.e} \quad I_2 = I_1\left[\frac{R_2}{R_2+R_3+R_4}\right] \quad \dots (2)$$

And

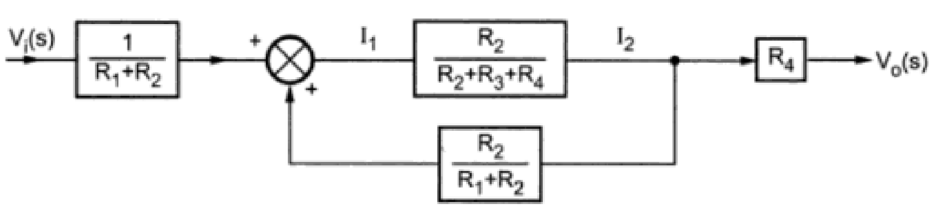
$$V_o = I_2 R_4 \quad \dots (3)$$

The block diagrams for the three equations are,



Equation (1) Equation (2) Equation (3)

Thus the overall block diagram is as shown in the Fig.



Ex.9. *A unity feedback system has* $G(s) = \dfrac{40\,(s+2)}{s\,(s+1)\,(s+4)}$.

Determine (i) Type of the system, (ii) All error coefficients and (iii) Error for ramp input with magnitude 4.

Solution : To determine type of system arrange G(s)H(s) in time constant form.

$$G(s)H(s) = \frac{40(s+2)}{s(s+1)(s+4)} = \frac{40(2)(1+0.5s)}{s(1+s)(4)(1+0.25s)}$$

$$= \frac{20(1+0.5s)}{s(1+s)(1+0.25\,s)}$$

Comparing this with standard form,

$$G(s)H(s) = \frac{K(1+T_1\,s)(1+T_2\,s)\ldots\ldots}{s^j\,(1+T_a\,s)(1+T_b\,s)\ldots\ldots}$$

where j = Type of system

∴ j = 1 so given system is type 1 system.

Error coefficients :

1) $$K_p = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \frac{20(1+0.5s)}{s(1+s)(1+0.25s)} = \infty$$

2) $$K_v = \lim_{s \to 0} s\,G(s)H(s) = \lim_{s \to 0} \frac{20(1+0.5s)}{(1+s)(1+0.25\,s)} = \mathbf{20}$$

3) $$K_a = \lim_{s \to 0} s^2\,G(s)H(s) = \lim_{s \to 0} \frac{s(1+0.5s)\cdot 20}{(1+s)(1+0.25s)} = \mathbf{0}$$

Now steady state error for ramp input is given by,

$$e_{ss} = \frac{A}{K_v}$$

where A = Magnitude of ramp input

Here A = 4 and K_v = 20

∴ $$e_{ss} = \frac{4}{20} = \mathbf{0.2}$$

Ex.10. : *For unity feedback system,*

$G(s) = \dfrac{K}{s(1+0.4s)(1+0.25s)}$ *, find range of values of K, marginal value of K and frequency of sustained oscillations.*

Solution : Characteristic equation, $1 + G(s)H(s) = 0$ and $H(s) = 1$

$$\therefore \quad 1 + \frac{K}{s(1+0.4s)(1+0.25s)} = 0$$

$$s[1 + 0.65s + 0.1s^2] + K = 0$$

$$\therefore \quad 0.1s^3 + 0.65s^2 + s + K = 0$$

s^3	0.1	1	From s^0, $K > 0$
s^2	0.65	K	From s^1,
s^1	$\frac{0.65 - 0.1K}{0.65}$	0	$0.65 - 0.1K > 0$ $\therefore 0.65 > 0.1K$
s^0	K		$\therefore 6.5 > K$

$\therefore$ Range of values of K, $0 < K < 6.5$.

The marginal value of 'K' is a value which makes any row other than s^0 as row of zeros.

$$\therefore \quad 0.65 - 0.1 K_{mar} = 0$$

$$\therefore \quad \boxed{K_{mar} = 6.5}$$

To find frequency, find out roots of auxiliary equation at marginal value of 'K'.

$$A(s) = 0.65s^2 + K = 0;$$

$$\therefore \quad 0.65s^2 + 6.5 = 0 \qquad \because K_{mar} = 6.5$$

$$s^2 = -10$$

$$s = \pm j\, 3.162$$

Comparing with $s = \pm j\omega$

ω = Frequency of oscillations

= **3.162 rad/sec.**

Ex.11 : *$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$. Check the stability of given characteristic equation using Routh's method.*

Solution :

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	0	0	0	0 ⇐ Special case 2

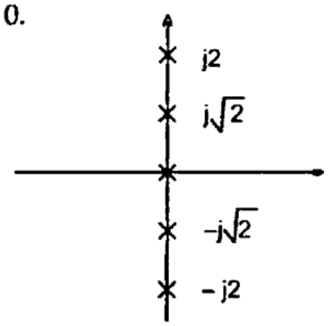
Row of zeros

$$A(s) = 2s^4 + 12s^2 + 16 = 0$$

$$\frac{dA}{ds} = 8s^3 + 24s = 0$$

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	8	24	0	0
s^2	6	16	0	
s^1	2.67	0		
s^0	16			

No sign change, so system may be stable. But as there is row of zero, system will be (i) marginally stable or (ii) unstable. To examine this solve A(s) = 0.



$$2s^4 + 12s^2 + 16 = 0$$

$$s^4 + 6s^2 + 8 = 0$$

Put $s^2 = y$

$\therefore \quad y^2 + 6y + 8 = 0$

$$y = -6 \pm \frac{\sqrt{36-32}}{2}$$

$$= -3 \pm 1 = -2, -4$$

$\therefore \quad s^2 = -2 \quad$ and $\quad s^2 = -4$

$\therefore \quad s = \pm j\sqrt{2} \quad$ and $\quad s = \pm j2$

Nonrepeated roots on imaginary axis. Hence system is marginally stable.