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1 DC Servo Motors

Many industries still prefer DC servo motors to AC motors for control applications due to their excellent performance. The attractive features of DC servo motors are high torque to inertia ratio, ease of speed control, capability in delivering more than three times their rated torque and so on. DC servo motors are classified as shown in Fig. .1.

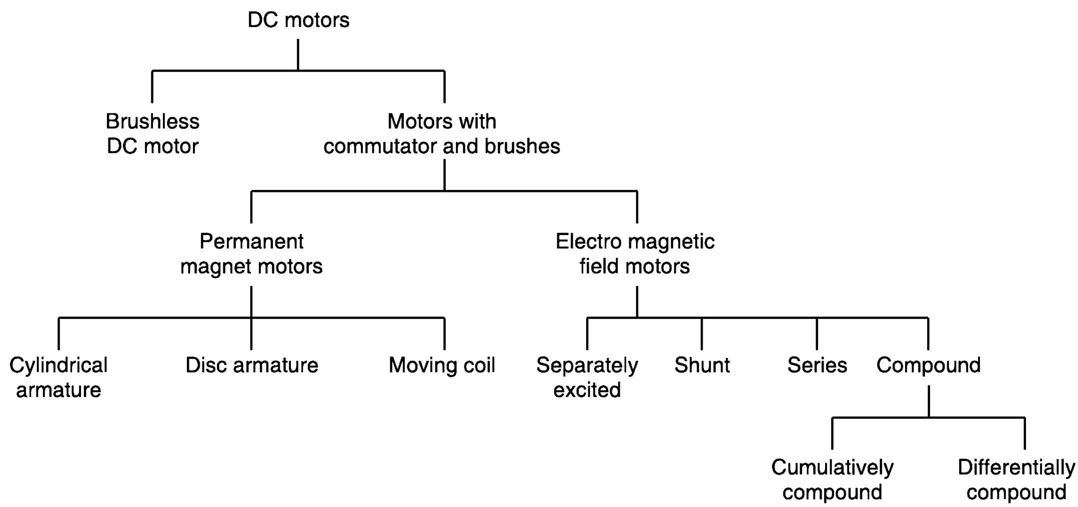


Fig. .1 **Classification of DC motor**

Brushless DC motors and permanent magnet motors were discussed in previous chapters. In this chapter, we discuss electromagnetic field motors, their working principles, characteristics, transfer function and control.

Electromagnetic field motors are economical for higher power ratings, generally above 1 kW. DC servo motors are similar to conventional DC motors in several aspects. They have the following special features:

- For improving commutation, a large number of slots and commutator segments are used.
- Compensating winding is used to reduce sparking.
- Inertia is reduced by keeping the ratio of diameter to length.
- Oversized shafts are used for withstanding high torque stress.
- To reduce hysteresis loss, low-loss steel is used in the magnetic circuit.
- The magnetic circuit is made of laminated steel to reduce eddy current loss.

1.1 Construction

DC servo motor has a stator and a rotor. The stator has an outer casing. The outer case has a hollow cylindrical shape. It is called the yoke or frame of the motor. A magnetic material is used for the yoke as it has to provide return path for the main flux. Yoke provides mechanical support for the poles. Yoke is covered by two end covers. The field system is stationary. It consists of pole core and winding. The coils of the field are wound in such a way over each pole that when they are excited by DC source, alternate north and south poles are developed in adjacent poles.

The armature is a rotating member. It is made up of circular sheet steel stampings. Along the outer periphery of the stampings, slots are provided. The armature conductors are housed in these slots. Closed windings are used for the armature. They have tappings and are connected to commutator segments. Both commutator and armature are mounted on the shaft. Commutator segments are insulated from each other and from the shaft by mica insulator. Brushes are used to connect DC source and armature with the help of commutator. The combined action of brushes and commutator converts the impressed DC voltage into AC voltage, as it is essential for the working of motor. The armature winding is divided into a number of parallel paths. The number of brushes and parallel paths are equal. The constructional details of a DC servo motor are shown in Fig. .2.

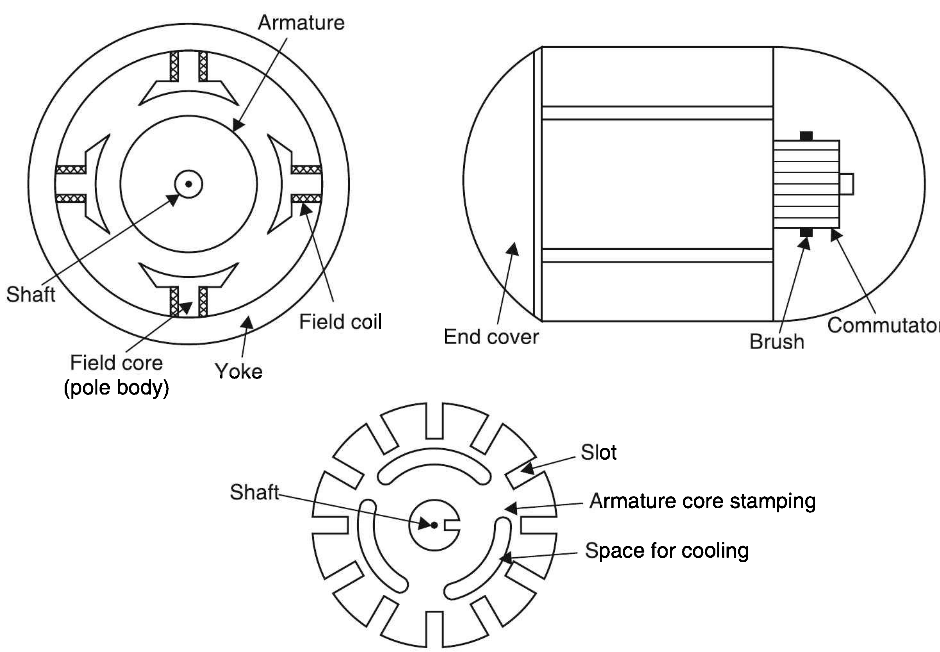


Fig .2 Constructional details of DC motor

Depending upon the type of motor, the field winding and armature windings are connected. In series motor, the field is connected in series with the armature and carries the same current. For controlling the current through the field, a variable resistance can be connected across the field. In shunt motor, the field and the armature are connected in parallel. In compound motors, there are two field windings; one is connected in series and other in parallel with the armature. In another type, which is commonly used in control system application, the field is not connected to the armature. The field is excited by a separate DC source. This type of motor is called separately excited DC motor. Configurations of various types of DC motors are shown in Fig .3.

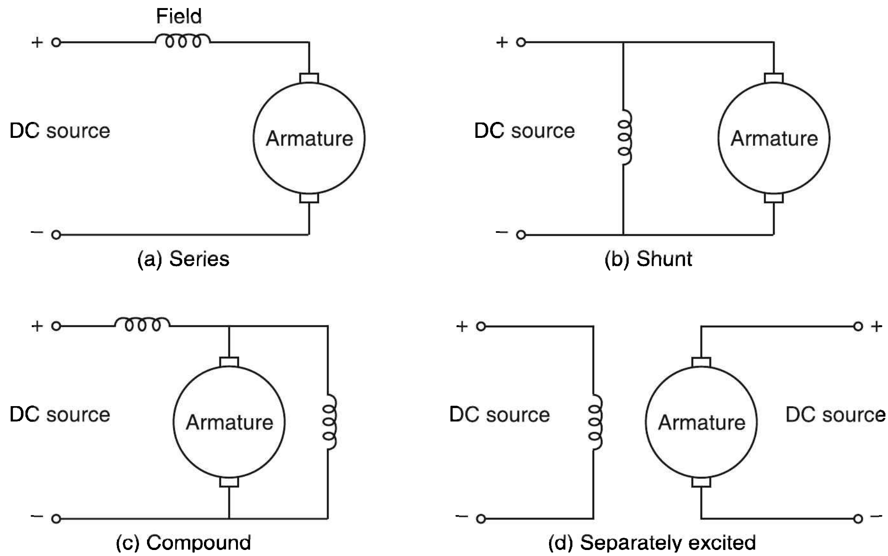


Fig. .3 Different types of DC motor

1.2 Principle of Operation

Like any other electrical motor DC, servo motor converts electrical energy into mechanical energy. It works on the basic principle that a current-carrying conductor placed in a magnetic field experiences a force, which is given by

$$F = BIl = \mu_o \mu_r HIl \text{ N-m} \qquad (.1)$$

where

B = flux density in Wb/m^2

I = current through the conductor

l = length of the conductor in the magnetic field

μ_o = permeability of free space

μ_r = relative permeability of the medium in which the conductor is placed

H = magnetic field strength, A/m^2

For a DC servo motor, the magnetic field is produced by the electromagnetic field system by execiting the winding. Armature has conductors which carry current when it is connected to the DC supply through the commutator and brushes. Consider a two-pole excited DC motor as shown in Fig. .4. Assume that the current flows through the armature conductors in the direction shown in Fig .4. By applying Flemings left-hand rule, it can be seen that the conductors experience forces in the directions shown in Fig .4. As the

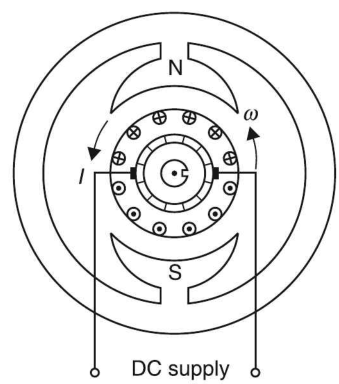


Fig .4 Working of DC servo motor

conductors are housed in the slots along the periphery of the armature, armature experiences a torque in the CCW direction and rotates in the CCW direction, provided the load torque is less than the net torque developed.

1.3 Voltage Equation of a DC Servo Motor and Characteristics

When the motor is running, the armature conductors cut the magnetic field and an emf is induced in the armature conductors due to change in flux linkage. This emf has a direction opposite to the supply voltage and hence it is known as back emf.

Let

P = number of poles

Z = number of armature conductor

ϕ = flux per pole, Wb

N = speed in rpm

A = number of parallel paths

The flux cut by an armature conductor/revolution,

$$d\phi = P\phi.$$

Number of revolution/sec, $dt = \dfrac{N}{60}$

Flux cut by an armature conductor/sec, $\dfrac{P\phi N}{60}$

∴ emf induced in one conductor, $\dfrac{d\phi}{dt} = \dfrac{P\phi N}{60}$

Number of conductor per parallel path, $\dfrac{Z}{A}$

∴ emf-induced/parallel path = emf induced in the armature across the brushes $\dfrac{P\phi N}{60} \cdot \dfrac{Z}{A}$

This emf is the back emf.

∴ $$E_b = \frac{\phi ZN}{60}\frac{P}{A} \quad (2)$$

For shunt and separately excited motors, the equivalent circuit may be drawn as in Fig. 5.

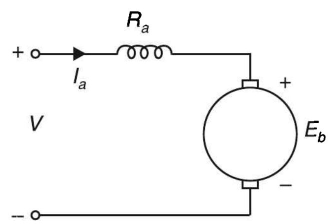


Fig. 5 Equivalent circuit of DC motor.

From the equivalent circuit, the voltage equation of the DC motor is given by

$$V = I_a R_a + E_b + V_{bb} \qquad (\ 3)$$

where V_{bb} is brush contact drop.

For series motor,

$$V = I_a R_a + I_a R_{se} + E_b + V_{bb}$$

where R_{se} is series field resistance.

1.4 Characteristics of DC Motor

The torque is developed by the interaction of the main field flux with the armature current and is given by

$$T \propto \phi I_a \qquad (.\ 4)$$

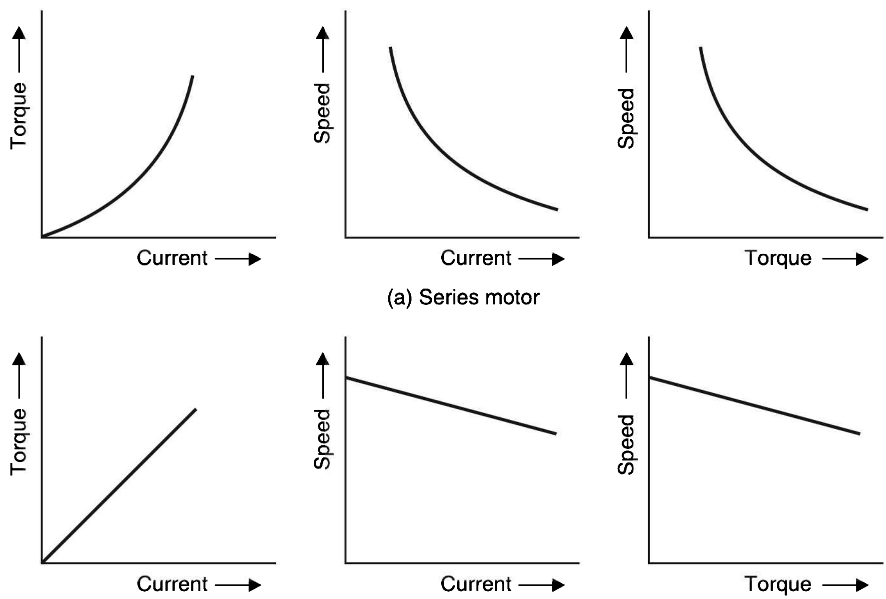
From Eq. (4), we get

$$E_b \propto \phi N \quad (Z, D, \text{and } A \text{ are constants for a given motor})$$

$$\therefore \qquad N \propto \frac{E_b}{\phi}$$

$$\therefore \qquad N \propto \frac{V - I_a R_a}{\phi}$$

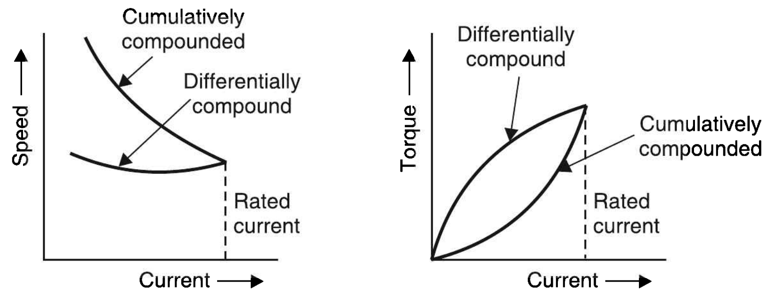
From the above relations, we can sketch the typical characteristics of DC servo motor. Characteristics of various types of DC motors are shown in Fig .6. DC shunt motor is a constant speed motor. DC series motor is a motor with starting torque.



(a) Series motor

(b) Shunt motor

Fig .6 *Contd...*



(c) Compound motor

Fig .6 Characteristics of DC motor

1.5 Transfer Function of DC Servo Motor

In servomechanisms and closed-loop control systems, DC motors are used in two modes: armature-controlled and field-controlled modes. Transfer functions of the DC motor in these two modes are explained in this section.

Let

$i_a(t)$ = armature current, A

$i_f(t)$ = field current, A

$v_f(t)$ = field supply voltage, V

$v_a(t)$ = armature supply voltage, V

$e_b(t)$ = back emf, V

$\phi(t)$ = flux/pole, Wb

$\omega(t)$ = angular velocity, rad/sec

$\theta(t)$ = angular position of shaft, rad

$T(t)$ = torque

J = moment of inertia of motor plus load

B = coefficient of friction of motor plus load

R_a = armature circuit resistance, Ω

L_a = armature inductance, H

R_f = field resistance, Ω

L_f = field inductance, H

K_t = torque constant

K_f = field constant

K_b = back emf constant

Transfer function of field-controlled DC motor

The schematic diagram of field-controlled DC servo motor is shown in Fig .7.

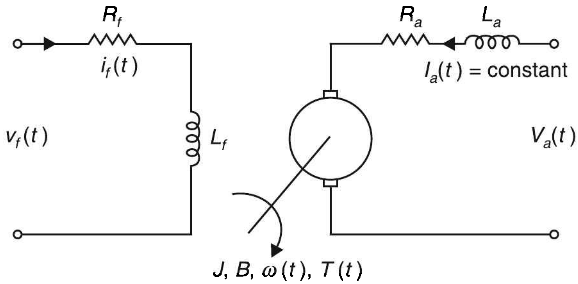


Fig .7 Field-controlled DC motor

In field-controlled mode, the armature current is kept constant and field circuit is controlled. The flux in the air gap is given by

$$\phi(t) = K_f i_f(t)$$

Torque is given by

$$T \propto \phi(t) I_a(t)$$

i.e.

$$T = K' K_f I_a(t) i_f(t)$$

$\therefore$

$$T = K i_f(t) \qquad (\;5)$$

where

$K' =$ constant

$K = K' K_f I_a(t)$

The field circuit voltage equation is

$$V_f(t) = R_f i_f(t) + L_f \frac{di_f(t)}{dt} \qquad (\;6)$$

The developed torque has to overcome the rotational moment of inertia and friction and is given by

$$T(t) = J\frac{d\omega(t)}{dt} + B\omega(t) \qquad (\;7)$$

Taking the Laplace transforms of Eqs. (3) to (5), we get

$$T(s) = K I_f(s) \qquad (\;8)$$

$$V_f(s) = (R_f + sL_f) I_f(s) \qquad (\;9)$$

$$T(s) = (Js + B)\Omega(s) \qquad (\;10)$$

and

$$\Theta(s) = \frac{\Omega(s)}{s} \qquad (\;11)$$

Equations (8) to (11) are used to develop the block diagram which is shown in Fig. .8.

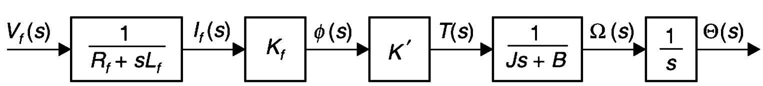


Fig .8 Block diagram of field-controlled DC motor

The transfer function of the field-controlled DC motor is given by

$$\frac{\Theta(s)}{V_f(s)} = \frac{K}{s(sL_f + R_f)(sJ + B)} \qquad (.12)$$

In field-controlled motor, the field current is varied, which is practically small in value. It is not economical to change small current is case of large motors. But this mode of control is required for speed control above the base speed of the motor.

Transfer function of armature-controlled DC motor

The schematic diagram of an armature-controlled DC motor is shown in Fig. .9.

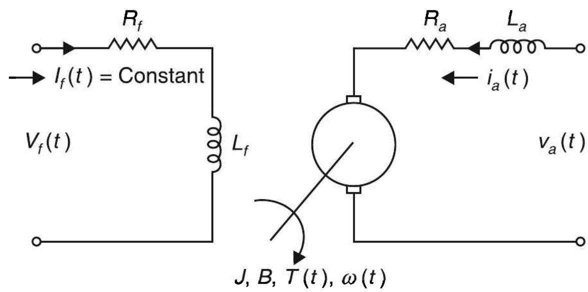


Fig .9 Armature-controlled DC motor

In armature-controlled mode, the field current is kept constant and armature current is varied by controlling the armature supply voltage. The air gap flux, $\phi(t) = K_f I_f(t)$, is a constant.

The torque developed is,

$$T(t) = K_t i_a(t) \qquad (13)$$

The back emf is given by

$$e_b(t) = K_b \omega(t) \qquad (14)$$

The applied armature voltage has to overcome the back emf, as voltage drops due to armature inductance and resistance. The voltage equation is given by,

$$v_a(t) = e_b(t) + R_a i_a(t) + L_a \frac{di_a(t)}{dt} \qquad (15)$$

The developed torque is used to accelerate the rotor against the rotational moment of inertia and friction.

$$\therefore \quad T(t) = J\frac{d\omega(t)}{dt} + B\omega(t) \qquad (16)$$

Also, $$\omega(t) = \frac{d\theta(t)}{dt} \qquad (17)$$

The Laplace transforms of Eqs. (13) to (7.17) are given by

$$T(s) = K_t I_a(s) \qquad (18)$$

$$E_b(s) = K_b \Omega(s) \qquad (.19)$$

$$V_a(s) = E_b(s) + R_a I_a(s) + sL_a I_a(s) \qquad (20)$$

$$T(s) = J_s \Omega(s) + B\Omega(s) \qquad (21)$$

$$\Omega(s) = s\theta(s) \qquad (22)$$

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Equations (18) to (22) are used to develop the block diagram of the armature-controlled motor as shown in Fig. .10.

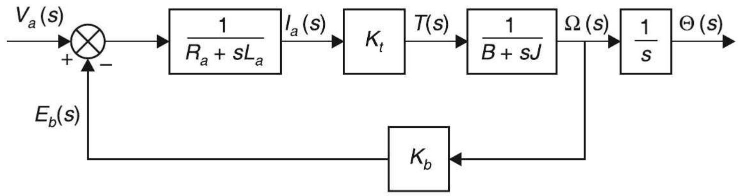

Fig. .10 Block diagram of armature controlled DC motor

The above block diagram can be reduced as shown in Fig. 11.

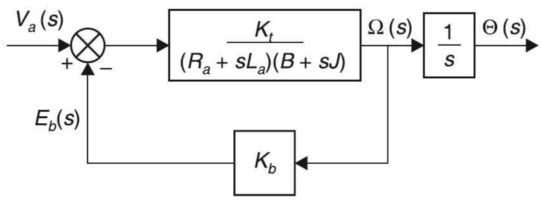

Fig. 11 Reduced block diagram

For the feedback loop

$$G(s) = \frac{K_t}{(R_a + sL_a)(B + sJ)}$$

$$H(s) = K_b$$

$$\therefore \quad \frac{\Omega(s)}{V_a(s)} = \frac{G(s)}{HG(J)H(s)} = \frac{K_t}{(R_a + sL_a)(B + sJ) + K_tK_b}$$

$$\therefore \quad \frac{\Theta(s)}{V_a(s)} = \frac{K_t}{s\left[K_tK_b + R_a\left(1 + \frac{sL_a}{R_a}\right) + B\left(1 + \frac{sJ}{B}\right)\right]}$$

i.e.

$$\frac{\Theta(s)}{V_a(s)} = \frac{K_t}{s[K_tK_b + R_a(1 + s\tau_1) + B(1 + s\tau_2)]} \qquad (\ .23)$$

where $\tau_1 = \frac{L_a}{R_a}$, electrical time constant

$\tau_2 = \frac{J}{B}$, mechanical time constant

1.6 Control of DC Servo Motor

For controlling the speed of DC servo motor, generally armature current control scheme is used. The armature current is controlled by varying the terminal voltage of the motor. Variable voltage for supplying the motor is obtained from an inverter consisting of solid state devices and control circuits. There are various schemes for implementing the controller. A few of them are described in this section.

A single-phase half controlled motor drive is shown in Fig. 12. The circuit is simple and low cost. A free-wheeling diode is used to improve the performance.

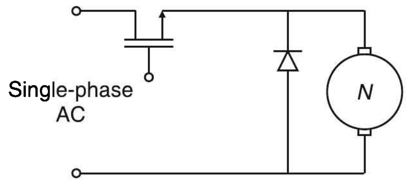

Fig. 12 Single-phase half controlled DC motor drive

This scheme has the following disadvantages:

- Only one quadrant operation is possible
- Regeneration is not possible
- At low speeds, oscillations in speed are present
- Loss of torque at rated current.

A fully controlled single-phase drive is shown in Fig. 13. In this configuration, regenerative braking is possible. The speed regulation is improved and oscillations in speed can be reduced.

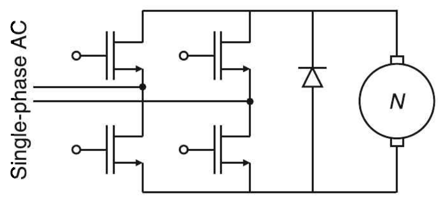

Fig. 13 Fully controlled DC motor drive

For large power ratings, three-phase drives are used. The commutation is better in three-phase drives compared with single-phase drives. Half controlled and fully controlled three-phase drives are shown in Figs. 14a and b respectively.

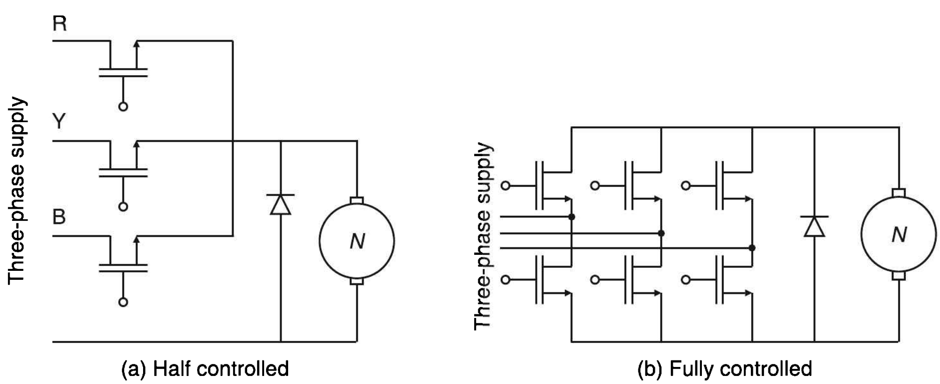

Fig. 14 Three-phase drives

For four quadrant operation, dual converter is used. The scheme is shown in Fig. .15.

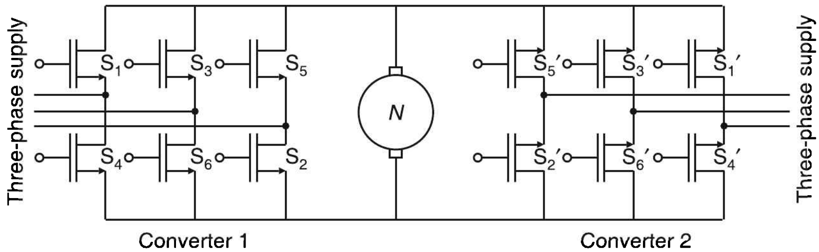

Fig. 15 Dual three-phase converter for DC drive

This scheme uses two three-phase-controlled bridge converters as shown in Fig. 15. Converter 1 is forward converter which supplies positive current for the first and the second quadrant operations. Converter 2 supplies current during the third and the fourth quadrant operations. The complete control scheme is shown in Fig. 16 as a block diagram. This scheme is suitable for both separtely excited and permanent magnet DC servo motors.

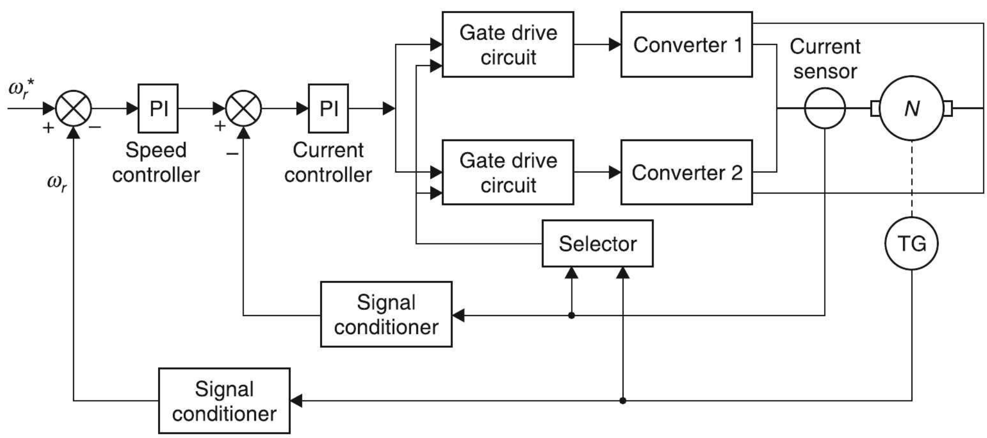

Fig. 16 Block diagram of a four quadrant, dual converter fed DC motor drive

A dual three-phase converter is used to supply the armature of the DC motor. The converter to be operated is chosen by a selector after processing the current, speed and zero crossing of current. Converter 1 is selected for the first and the second quadrant operations and converter 2 for the third and the fourth quadrant operations. A tachogenerator (TG) is used in the speed feedback loop and a current sensor is used for current loop. PI controllers are used in the speed and current loops.

If the source is DC, chopper circuits are used for converting fixed supply voltage into variable voltage. In chopper circuits, the switching devices are turned OFF and ON for specified durations to control the variable voltage. The output voltage of a chopper is given by

$$V_o = \frac{t_{ON}}{t_{ON} + t_{OFF}} V_i$$

where

V_i = input DC voltage

t_{ON} = time for which the switch is ON

t_{OFF} = time for which the switch is OFF

The ratio $\frac{t_{ON}}{t_{ON} + t_{OFF}}$ is called duty cycle.

By varying the duty cycle, the output voltage of the chopper is controlled. A closed-loop control scheme for chopper-fed DC servo motor is shown in Fig. 17. It uses an inner current loop and outer speed loop. The current controller can be of PWM (pulse width modulation) type or hysteresis type. A PI controller is used as a speed controller.

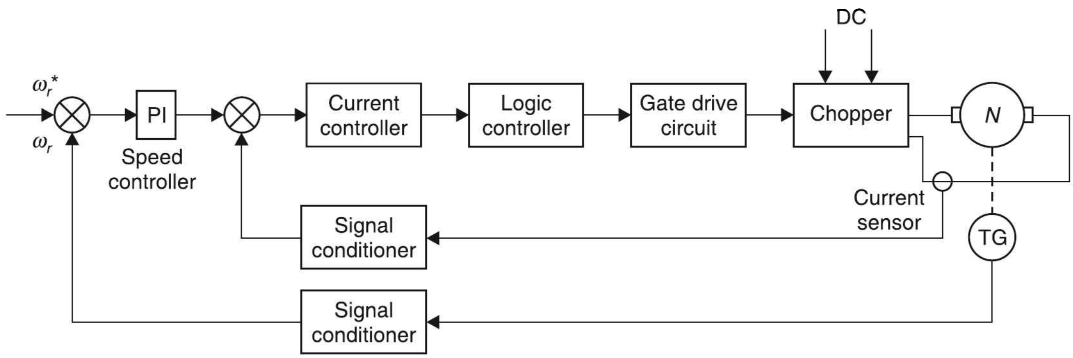

Fig. 17 **Closed-loop control scheme for a chopper-fed DC motor**

The current controller based on PWM technique checks the quadrant in which the motor is operating and decides the appropriate switching devices to be operated. This is done by checking the polarities of current and control voltage and directions. The ON time is decided by comparing a ramp signal with control voltage. The chopper circuit for four quadrant operations is shown in Fig. 18.

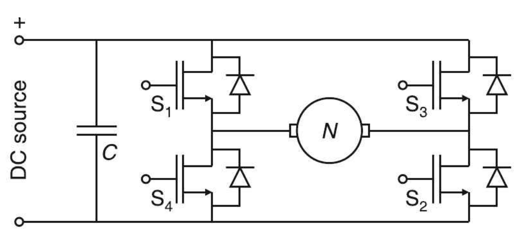

Fig. 18 **Chopper power circuit for four quadrant operation of DC motor**

Microprocessor-based control of DC servo motor

The microprocessor-based control scheme for a dual converter-fed DC servo motor is shown in Fig. 19. The microprocessor gets its inputs current, speed, synchronising signal as digital signals. The controllers are software implemented. It provides signals for operating the appropriate switching devices as digital signals. These signals are converted into analogue signals and given to the gates after proper amplification by the gate drive circuit.

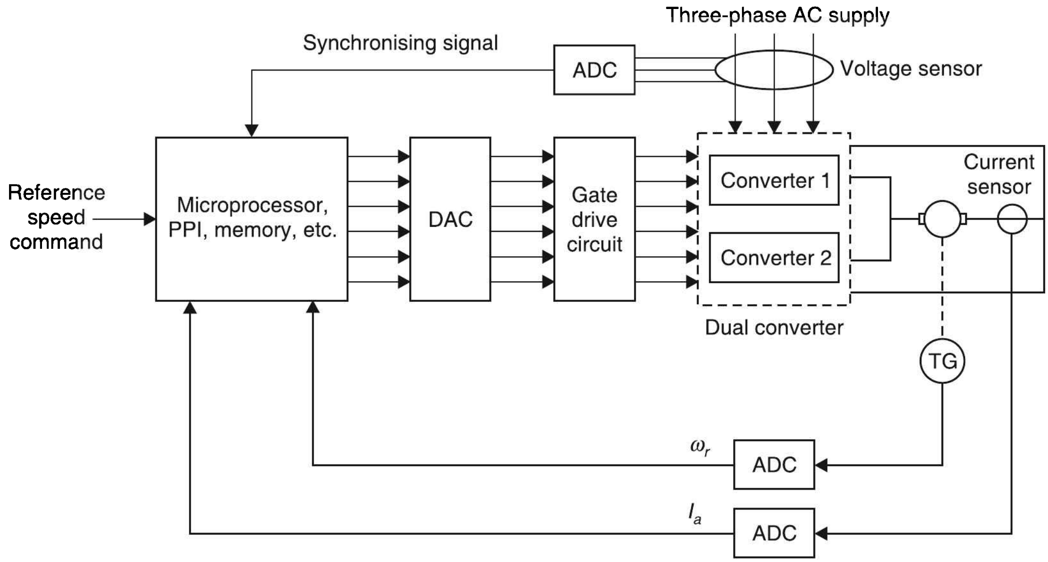

Fig. 19 Microprocessor-based control scheme for DC servo motor

The speed–torque characteristics of armature-controlled separately excited and series DC servo motors are shown in Fig. 20.

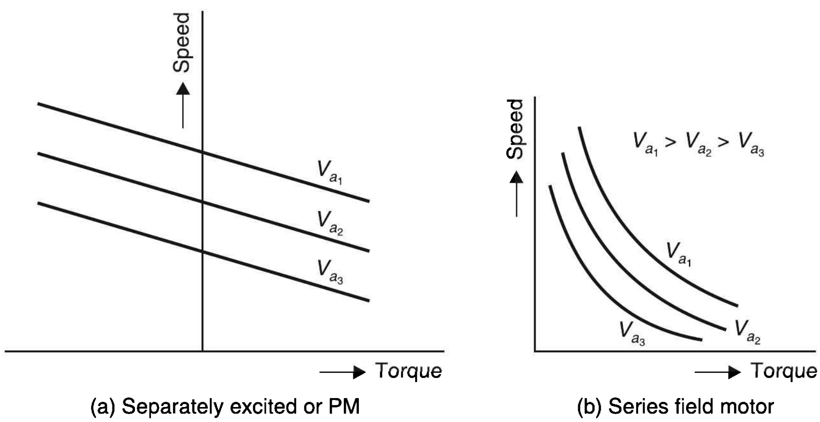

Fig. 20 Torque–speed characteristics of armature-controlled DC servo motors

DC Servo Motors (Cont)

CONTROL OF DC MOTORS

Control of dc motors is accomplished by using SCRs to modulate the input voltages to the armature and/or the field circuit of the motor. For an ac source, phase-controlled rectifiers are employed; for a dc source, choppers. But before we discuss these, it is worthwhile to consider the analysis of some passive RL-circuits involving diodes or SCRs.

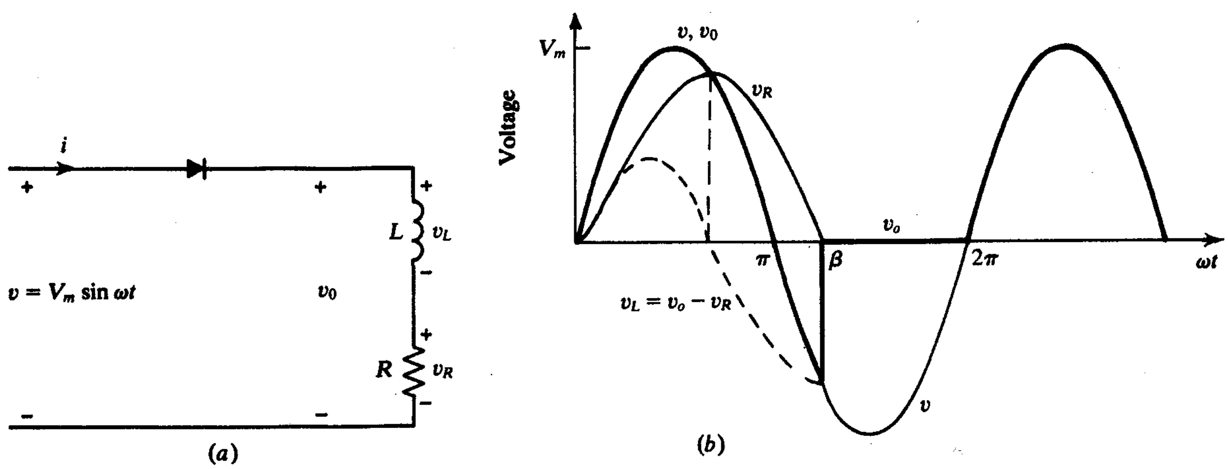

Fig. 1

1. *Half-wave rectifier with RL-load.* A half-wave rectifier with *RL*-load is indicated in Fig. 1(*a*). It may be shown [illegible] that the current during one period of the applied voltage $v = V_m \sin \omega t$ is given by

$$i = \begin{cases} \dfrac{V_m}{Z} [\sin (\omega t - \phi) + e^{-(R/L)t} \sin \phi] & 0 < \omega t < \beta \\ 0 & \beta < \omega t < 2\pi \end{cases} \qquad (1)$$

where

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \tan\phi = \frac{\omega L}{R} \tag{2}$$

The *extinction time* (the time at which the diode stopped conducting) is β/ω; and β can be found from the condition that i be continuous at the extinction time. Thus:

$$\sin(\beta - \phi) + e^{-\beta\cot\phi}\sin\phi \pm 0 \tag{3}$$

a transcendental equation for β. The average value of $i(t)$ over a period $2\pi/\omega$ is

$$I_{avg} = \frac{V_m}{2\pi R}(1 - \cos\beta) \tag{4}$$

Because the average voltage across the inductor is zero, the average voltage across the load is given by

$$V_{o\,avg} = V_{R\,avg} = RI_{avg} = \frac{V_m}{2\pi}(1 - \cos\beta) \tag{5}$$

The four voltage waveforms are shown in Fig. 1.(*b*)

2. *Half-wave rectifier with dc-motor load.* The circuit is shown in Fig. 2(*a*), where R and L are respectively the armature-circuit resistance and inductance, and e' is the motor back emf, assumed constant. The circuit analysis leads to the following expression for the current:

$$i = \begin{cases} 0 & 0 < \omega t < \alpha \\ \dfrac{V_m}{Z}\left[\sin(\omega t - \phi) + Be^{-(R/L)t}\right] - \dfrac{e'}{R} & \alpha < \omega t < \beta \\ 0 & \beta < \omega t < 2\pi \end{cases} \tag{6}$$

where

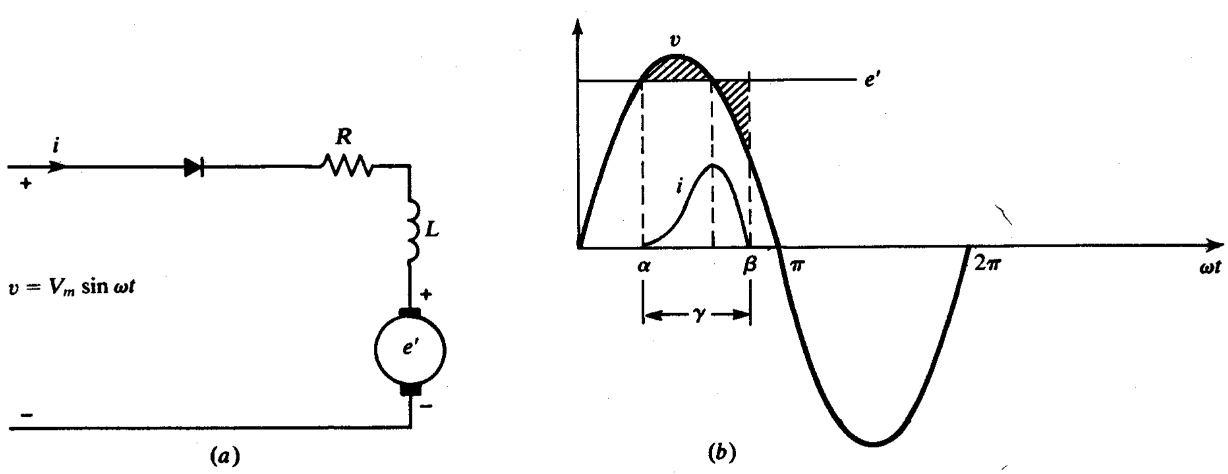

Fig. 2

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \tan\phi = \frac{\omega L}{R}$$

and where

$$B \equiv \left[\frac{e'}{V_m \cos\phi} - \sin(\alpha - \phi)\right] e^{\alpha R/\omega L} \tag{7}$$

is such as to make i continuous at $\omega t = \alpha$. It is seen from (6) that the diode starts conducting at $\omega t = \alpha$; the *firing angle*, α, is determined by the condition $v = e' + 0$, i.e.,

$$\sin\alpha = \frac{e'}{V_m} \tag{8}$$

As is shown in Fig. 2 (b), conduction does not necessarily stop when v becomes less than e'; rather, it ends at $\omega t = \beta$, when the energy stored in the inductor during the current buildup has been completely recovered. The *extinction angle*, β, may be determined from the continuity of 6 at $\omega t = \beta$; we find

$$\sin(\beta - \phi) + Be^{-\beta\cot\phi} = \frac{\sin\alpha}{\cos\phi} \tag{9}$$

as the transcendental equation for β, in which B is known from 6 . The average value of the current over one period of the applied voltage is found to be

$$I_{avg} = \frac{1}{R} V_{Ravg} = \frac{V_m}{2\pi R}(\cos\alpha - \cos\beta - \gamma\sin\alpha) \tag{10}$$

where $\gamma \equiv \beta - \alpha$ is the *conduction angle*. Figure 2 (b) shows the waveforms.

SCR-Controlled DC Motor

In the example above, the dc-motor load was not controlled by the half-wave rectifier; the back emf remained constant, implying that the motor speed was unaffected by the cyclic firing and extinction of the diode. To achieve a control, we use a thyristor instead of the diode, as shown in Fig. 3(a). The corresponding waveforms are illustrated in Fig. 3(b). The motor torque (or speed) may be varied by varying α. Explicitly, for the armature we integrate

$$v_m = Ri + L\frac{di}{dt} + e \tag{11}$$

over the conduction period $\alpha/\omega < t < \beta/\omega$, during which v_m coincides with the line voltage v. The result is:

$$V_m' = \frac{V_m(\cos\alpha - \cos\beta)}{\gamma} = RI' + E' \tag{12}$$

where a prime indicates an average over the conduction period. Over a full period of the line voltage, the average armature current is given by

$$I_{avg} = \frac{\gamma}{2\pi} I' \tag{13}$$

and the average torque is given by

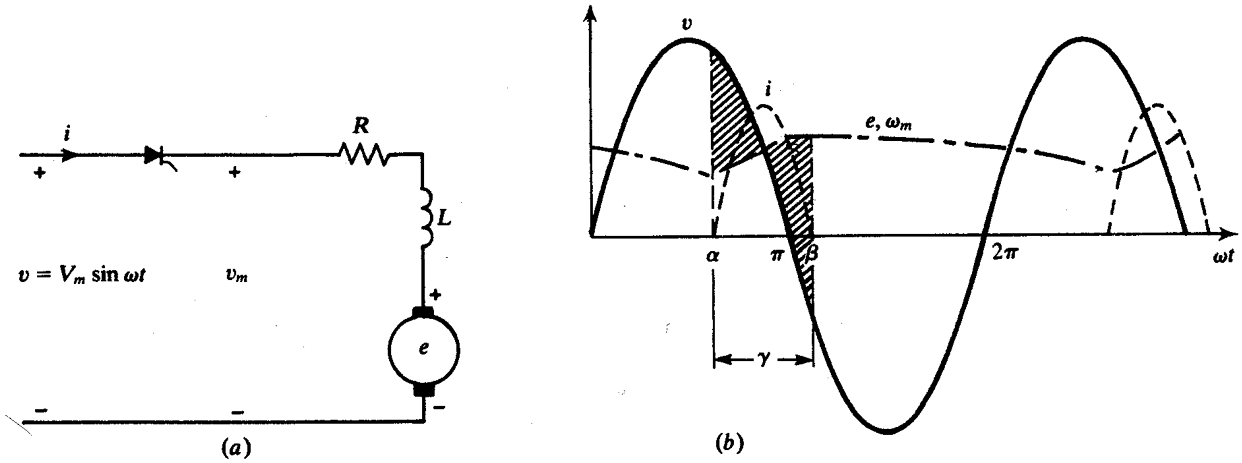

Fig. 3

$$T_{avg} = kI_{avg} = \frac{k\gamma}{2\pi} I' \qquad (14)$$

Equations 12, 13, and 14 govern the steady-state performance of a thyristor-controlled dc motor.

P1:A dc motor having an armature resistance of 0.51 Ω and an inductance of 0.78 mH is connected in series with a diode to a 110-V, 60-Hz ac source. The motor is running at an essentially constant speed of 970 rpm. The motor back-emf constant has the value 0.08 V/rpm. Determine the average value of the armature current.

The desired current is given by (10), with a determined from 8 and β from (9). A graphical solution of 9 is presented in Fig. (4), where $m = \sin\alpha$ and $\gamma = \beta - \alpha$. From the data,

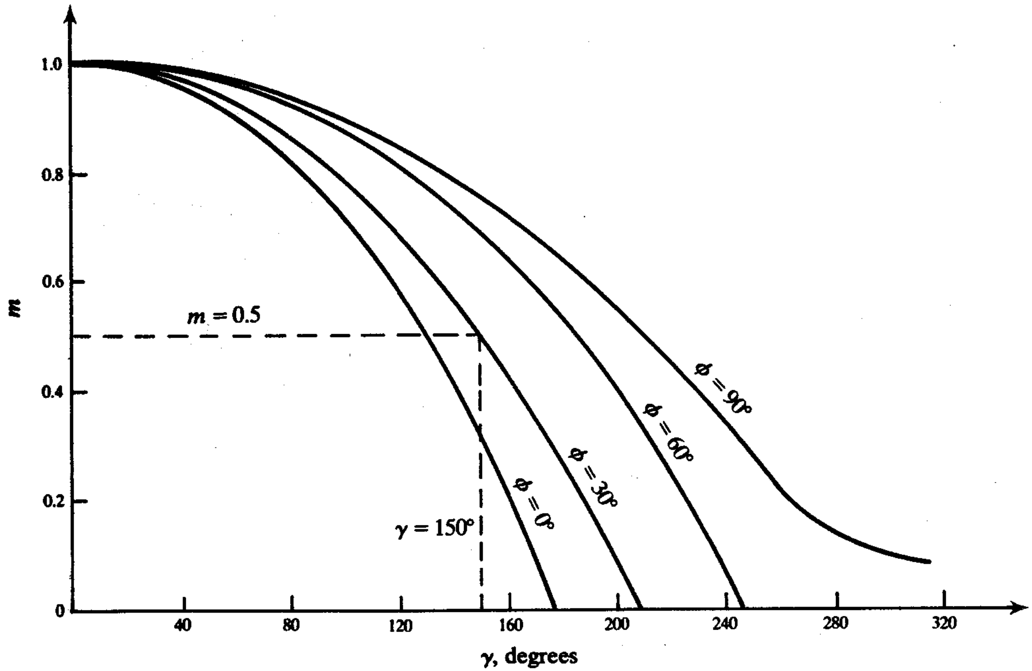

Fig. 4

$$\tan\phi = \frac{\omega L}{R} = \frac{(120\pi)(0.78 \times 10^{-3})}{0.51} = 0.577 \qquad \text{or} \qquad \phi = 30°$$

$$\sin\alpha = \frac{e'}{V_m} = \frac{(0.08)(970)}{110\sqrt{2}} = 0.5 \qquad \text{or} \qquad \alpha = 30°$$

Hence, from Fig. .4 , $\gamma = 150°$ and $\beta = \alpha + \gamma = 180°$. Then:

$$I_{avg} = \frac{110\sqrt{2}}{2\pi(0.51)}\left[\cos 30° - \cos 180° - \left(\frac{150}{180}\pi\right)(0.5)\right] = 27 \text{ A}$$

P2: Find the average torque developed by the motor of Problem

$$T_{avg} = kI_{avg} = \left(\frac{0.08}{2\pi/60}\right)(27) = 20.6 \text{ N} \cdot \text{m}$$