Ans Q#1:

flat perfectly conducting surface in xy plane is situated in a magnetic field,

$$\overline{\mathbf{H}} = 3\cos x\,\bar{a}_x + z\cos x\,\bar{a}_y \text{ A/m for } z \geq 0$$

$$= 0 \text{ for } z < 0$$

Find the current density on the conductor surface.

Sol. : From the point form of Ampere's circuit law,

$$\nabla \times \overline{\mathbf{H}} = \bar{\mathbf{J}} = \text{current density}$$

$$\therefore \quad \bar{\mathbf{J}} = \begin{vmatrix} \bar{\mathbf{a}}_x & \bar{\mathbf{a}}_y & \bar{\mathbf{a}}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} \text{ in cartesian form}$$

$$= \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right]\bar{\mathbf{a}}_x + \left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right]\bar{\mathbf{a}}_y + \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]\bar{\mathbf{a}}_z$$

From $\overline{\mathbf{H}}$, $\quad H_x = 3\cos x, \quad H_y = z\cos x, \quad H_z = 0$

$$\therefore \quad \bar{\mathbf{J}} = \left[0 - \frac{\partial z\cos x}{\partial z}\right]\bar{\mathbf{a}}_x + \left[\frac{\partial 3\cos x}{\partial z} - 0\right]\bar{\mathbf{a}}_y$$

$$+ \left[\frac{\partial z\cos x}{\partial x} - \frac{\partial 3\cos x}{\partial y}\right]\bar{\mathbf{a}}_z$$

$$= -\cos x\,\bar{\mathbf{a}}_x + 0\,\bar{\mathbf{a}}_y + 0\,\bar{\mathbf{a}}_z = -\cos x\,\bar{\mathbf{a}}_x \text{ A/m}^2$$

Thus, $\quad \bar{\mathbf{J}} = -\cos x\,\bar{\mathbf{a}}_x \text{ A/m}^2 \quad$ ... For $z \geq 0$

$\quad = 0 \text{ A/m}^2 \quad$ ... For $z < 0$

Ans_Q#2:

If a particular field is given by,

$$\bar{F} = (x + 2y + az)\bar{a}_x + (bx - 3y - z)\bar{a}_y + (4x + cy + 2z)\bar{a}_z$$

then find the constants a, b and c such that the field is irrotational.

Sol. :

Key Point: *The vector field is irrotational if its curl is zero.*

$$\therefore \quad \nabla \times \bar{F} = 0 \quad \text{... For } \bar{F} \text{ to be irrotational}$$

$$\text{Now} \quad \nabla \times \bar{F} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \left[\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right]\bar{a}_x + \left[\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right]\bar{a}_y + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right]\bar{a}_z = 0$$

$$\therefore \quad \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} = \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial z} = \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = 0$$

And $F_x = x + 2y + az$, $F_y = bx - 3y - z$, $F_z = 4x + cy + 2z$

$$\therefore \quad \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} = c - 1 = 0 \quad \text{i.e. } c = 1$$

$$\therefore \quad \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} = a - 4 = 0 \quad \text{i.e. } a = 4$$

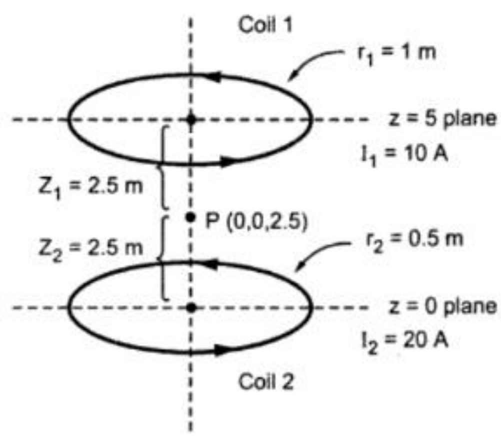
$$\therefore \quad \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = b - 2 = 0 \quad \text{i.e. } b = 2$$

Thus a = 4, b= 2 and c = 1 to have $\bar{F}$ irrotational.

Ans Q#3:

Two circular coils are located at the z = 0 plane and z = 5 m plane, centered about the z axis. The first coil has radius 1m and carries current of 10 A while second coil has radius 0.5 m carries current of 20 A. Calculate the magnetic field intensity at (0, 0, 2.5 m).

Sol. : The coils are shown in the Fig.



Assuming direction of I_1 and I_2 same, according to right hand rule $\overline{H}_1$ and $\overline{H}_2$ at P due to coil 1 and coil 2 are in same direction i.e. $\bar{a}_z$ direction.

$$\overline{H}_1 = \frac{I_1 r_1^2}{2\left(r_1^2 + z_1^2\right)^{\frac{3}{2}}} \bar{a}_z$$

$$= \frac{10\times(1)^2}{2\left[1^2+2.5^2\right]^{\frac{3}{2}}}\bar{\mathbf{a}}_z$$

$$= 0.2561\ \bar{\mathbf{a}}_z$$

and

$$\overline{\mathbf{H}}_2 = \frac{I_2 r_2^2}{2\left(r_2^2+z_2^2\right)^{\frac{3}{2}}}\bar{\mathbf{a}}_z$$

$$= \frac{20\times(0.5)^2}{2\left[0.5^2+2.5^2\right]^{\frac{3}{2}}}\bar{\mathbf{a}}_z$$

$$= 0.1508\ \bar{\mathbf{a}}_z$$

$\therefore$

$$\overline{\mathbf{H}} = \overline{\mathbf{H}}_1 + \overline{\mathbf{H}}_2$$

$$= 0.2561\ \bar{\mathbf{a}}_z + 0.1508\ \bar{\mathbf{a}}_z$$

$$= 0.4069\ \bar{\mathbf{a}}_z\ \text{A/m}$$