Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

Magnetic Boundary Conditions

Problem-1: The x–y plane separates two magnetic media with magnetic permeabilities μ_1 and μ_2, as shown in Fig.1. If there is no surface current at the interface and the magnetic field in medium 1 is

$$\mathbf{H}_1 = \hat{\mathbf{x}}H_{1x} + \hat{\mathbf{y}}H_{1y} + \hat{\mathbf{z}}H_{1z},$$

find:

(a) $\mathbf{H}_2$,

(b) θ_1 and θ_2, and

(c) evaluate $\mathbf{H}_2$, θ_1, and θ_2 for $H_{1x} = 2$ (A/m), $H_{1y} = 0$, $H_{1z} = 4$ (A/m), $\mu_1 = \mu_0$, and $\mu_2 = 4\mu_0$.

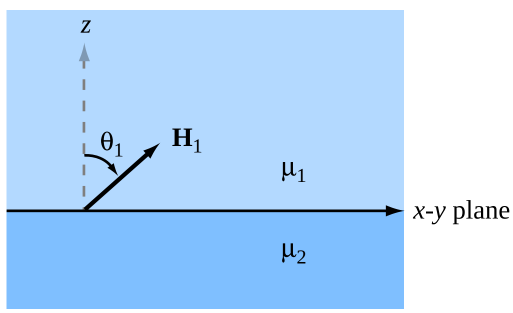

Fig.1: Adjacent magnetic media.

Solution:

(a) From,

$$\mu_1 H_{1\mathrm{n}} = \mu_2 H_{2\mathrm{n}},$$

and in the absence of surface currents at the interface,

$$H_{1\mathrm{t}} = H_{2\mathrm{t}}.$$

In this case, $H_{1z} = H_{1\mathrm{n}}$, and H_{1x} and H_{1y} are tangential fields. Hence,

$$\mu_1 H_{1z} = \mu_2 H_{2z},$$
$$H_{1x} = H_{2x},$$
$$H_{1y} = H_{2y},$$

Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

and

$$\mathbf{H}_2 = \hat{\mathbf{x}} H_{1x} + \hat{\mathbf{y}} H_{1y} + \hat{\mathbf{z}} \frac{\mu_1}{\mu_2} H_{1z}.$$

(b)

$$H_{1t} = \sqrt{H_{1x}^2 + H_{1y}^2},$$
$$\tan\theta_1 = \frac{H_{1t}}{H_{1z}} = \frac{\sqrt{H_{1x}^2 + H_{1y}^2}}{H_{1z}},$$
$$\tan\theta_2 = \frac{H_{2t}}{H_{2z}} = \frac{\sqrt{H_{1x}^2 + H_{1y}^2}}{\frac{\mu_1}{\mu_2} H_{1z}} = \frac{\mu_2}{\mu_1} \tan\theta_1.$$

(c)

$$\mathbf{H}_2 = \hat{\mathbf{x}} 2 + \hat{\mathbf{z}} \frac{1}{4} \cdot 4 = \hat{\mathbf{x}} 2 + \hat{\mathbf{z}} \quad \text{(A/m)},$$
$$\theta_1 = \tan^{-1}\left(\frac{2}{4}\right) = 26.56^\circ,$$
$$\theta_2 = \tan^{-1}\left(\frac{2}{1}\right) = 63.44^\circ.$$

Problem-2: Given that a current sheet with surface current density $\mathbf{J}_s = \hat{\mathbf{x}} 8$ (A/m) exists at $y = 0$, the interface between two magnetic media, and $\mathbf{H}_1 = \hat{\mathbf{z}} 11$ (A/m) in medium 1 $(y > 0)$, determine $\mathbf{H}_2$ in medium 2 $(y < 0)$.

Solution:

$$\mathbf{J}_s = \hat{\mathbf{x}} 8 \text{ A/m},$$
$$\mathbf{H}_1 = \hat{\mathbf{z}} 11 \text{ A/m}.$$

$\mathbf{H}_1$ is tangential to the boundary, and therefore $\mathbf{H}_2$ is also. With $\hat{\mathbf{n}}_2 = \hat{\mathbf{y}}$, we have

$$\hat{\mathbf{n}}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s,$$
$$\hat{\mathbf{y}} \times (\hat{\mathbf{z}} 11 - \mathbf{H}_2) = \hat{\mathbf{x}} 8,$$
$$\hat{\mathbf{x}} 11 - \hat{\mathbf{y}} \times \mathbf{H}_2 = \hat{\mathbf{x}} 8,$$

or

$$\hat{\mathbf{y}} \times \mathbf{H}_2 = \hat{\mathbf{x}} 3,$$

Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

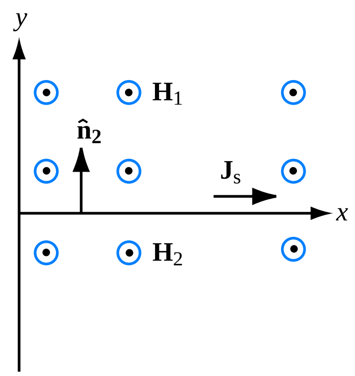

Fig.2: Adjacent magnetic media with $\mathbf{J}_s$ on boundary.

which implies that $\mathbf{H}_2$ does not have an x-component. Also, since $\mu_1 H_{1y} = \mu_2 H_{2y}$ and $\mathbf{H}_1$ does not have a y-component, it follows that $\mathbf{H}_2$ does not have a y-component either. Consequently, we conclude that

$$\mathbf{H}_2 = \hat{\mathbf{z}}\,3.$$

Problem-1: In Fig. 3, the plane defined by $x - y = 1$ separates medium 1 of permeability μ_1 from medium 2 of permeability μ_2. If no surface current exists on the boundary and

$$\mathbf{B}_1 = \hat{\mathbf{x}}2 + \hat{\mathbf{y}}3 \quad \text{(T)},$$

find $\mathbf{B}_2$ and then evaluate your result for $\mu_1 = 5\mu_2$. Hint: Start out by deriving the equation for the unit vector normal to the given plane.

Solution: We need to find $\hat{\mathbf{n}}_2$. To do so, we start by finding any two vectors in the plane $x - y = 1$, and to do that, we need three non-collinear points in that plane. We choose $(0,-1,0)$, $(1,0,0)$, and $(1,0,1)$.

Vector $\mathbf{A}_1$ is from $(0,-1,0)$ to $(1,0,0)$:

$$\mathbf{A}_1 = \hat{\mathbf{x}}\,1 + \hat{\mathbf{y}}\,1.$$

Vector $\mathbf{A}_2$ is from $(1,0,0)$ to $(1,0,1)$:

$$\mathbf{A}_2 = \hat{\mathbf{z}}\,1.$$

Hence, if we take the cross product $\mathbf{A}_2 \times \mathbf{A}_1$, we end up in a direction normal to the given plane, from medium 2 to medium 1,

$$\hat{\mathbf{n}}_2 = \frac{\mathbf{A}_2 \times \mathbf{A}_1}{|\mathbf{A}_2 \times \mathbf{A}_1|} = \frac{\hat{\mathbf{z}}\,1 \times (\hat{\mathbf{x}}\,1 + \hat{\mathbf{y}}\,1)}{|\mathbf{A}_2 \times \mathbf{A}_1|} = \frac{\hat{\mathbf{y}}\,1 - \hat{\mathbf{x}}\,1}{\sqrt{1+1}} = \frac{\hat{\mathbf{y}}}{\sqrt{2}} - \frac{\hat{\mathbf{x}}}{\sqrt{2}}.$$

Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

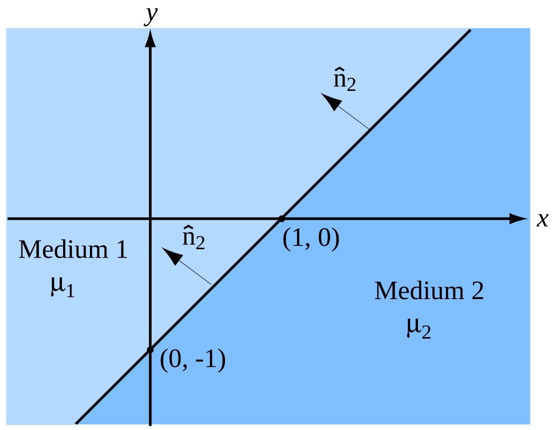

Fig.3: Magnetic media separated by the plane $x - y = 1$

In medium 1, normal component is

$$B_{1n} = \hat{\mathbf{n}}_2 \cdot \mathbf{B}_1 = \left(\frac{\hat{\mathbf{y}}}{\sqrt{2}} - \frac{\hat{\mathbf{x}}}{\sqrt{2}}\right) \cdot (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}3) = \frac{3}{\sqrt{2}} - \frac{2}{\sqrt{2}} = \frac{1}{\sqrt{2}},$$

$$\mathbf{B}_{1n} = \hat{\mathbf{n}}_2 B_{1n} = \left(\frac{\hat{\mathbf{y}}}{\sqrt{2}} - \frac{\hat{\mathbf{x}}}{\sqrt{2}}\right) \cdot \frac{1}{\sqrt{2}} = \frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2}.$$

Tangential component is

$$\mathbf{B}_{1t} = \mathbf{B}_1 - \mathbf{B}_{1n} = (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}3) - \left(\frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2}\right) = \hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5.$$

Boundary conditions:

$$B_{1n} = B_{2n}, \quad \text{or} \quad \mathbf{B}_{2n} = \frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2},$$

$$H_{1t} = H_{2t}, \quad \text{or} \quad \frac{B_{2t}}{\mu_2} = \frac{B_{1t}}{\mu_1}.$$

Hence,

$$\mathbf{B}_{2t} = \frac{\mu_2}{\mu_1}\mathbf{B}_{1t} = \frac{\mu_2}{\mu_1}(\hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5).$$

Finally,

$$\mathbf{B}_2 = \mathbf{B}_{2n} + \mathbf{B}_{2t} = \left(\frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2}\right) + \frac{\mu_2}{\mu_1}(\hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5).$$

For $\mu_1 = 5\mu_2$,

$$\mathbf{B}_2 = \hat{\mathbf{y}} \quad \text{(T)}.$$

Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

Inductance and Magnetic Energy

Problem-1: Obtain an expression for the self-inductance per unit length for the parallel wire transmission line of Fig. 1(a) in terms of a, d, and μ, where a is the radius of the wires, d is the axis-to-axis distance between the wires, and μ is the permeability of the medium in which they reside.

Solution: Let us place the two wires in the x–z plane and orient the current in one of them to be along the $+z$-direction and the current in the other one to be along the $-z$-direction, the magnetic field at point $P(x,0,z)$ due to wire 1 is

$$\mathbf{B}_1 = \hat{\boldsymbol{\phi}} \frac{\mu I}{2\pi r} = \hat{\mathbf{y}} \frac{\mu I}{2\pi x},$$

where the permeability has been generalized from free space to any substance with permeability μ, and it has been recognized that in the x-z plane, $\hat{\boldsymbol{\phi}} = \hat{\mathbf{y}}$ and $r = x$ as long as $x > 0$.

Given that the current in wire 2 is opposite that in wire 1, the magnetic field created by wire 2 at point $P(x,0,z)$ is in the same direction as that created by wire 1, and it is given by

$$\mathbf{B}_2 = \hat{\mathbf{y}} \frac{\mu I}{2\pi(d-x)}.$$

Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

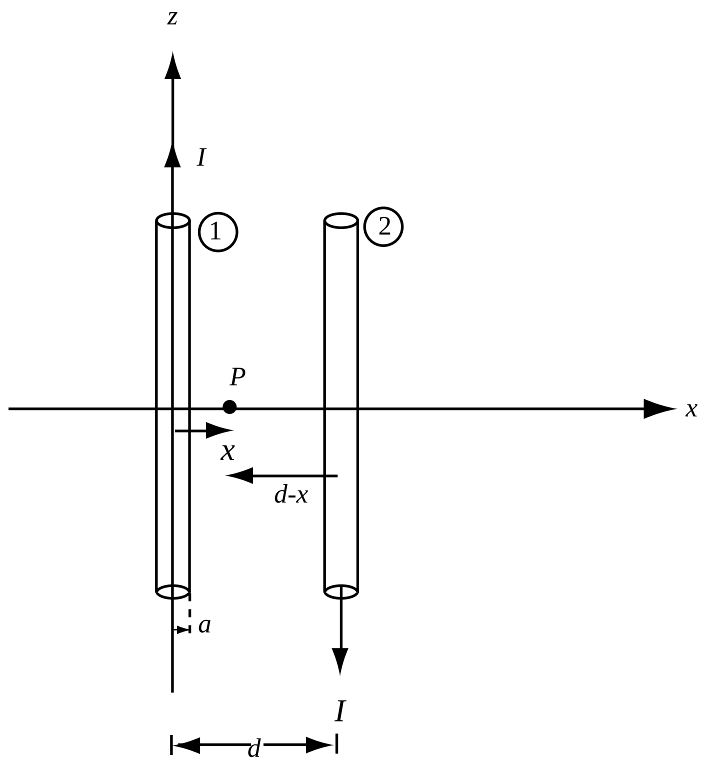

Fig.1: Parallel wire transmission line.

Therefore, the total magnetic field in the region between the wires is

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = \hat{\mathbf{y}}\frac{\mu I}{2\pi}\left(\frac{1}{x} + \frac{1}{d-x}\right) = \hat{\mathbf{y}}\frac{\mu I d}{2\pi x(d-x)}\,.$$

the flux crossing the surface area between the wires over a length l of the wire structure is

$$\begin{aligned}\Phi = \iint_S \mathbf{B}\cdot d\mathbf{s} &= \int_{z=z_0}^{z_0+l}\int_{x=a}^{d-a}\left(\hat{\mathbf{y}}\frac{\mu I d}{2\pi x(d-x)}\right)\cdot(\hat{\mathbf{y}}\,dx\,dz)\\ &= \frac{\mu I l d}{2\pi}\left(\frac{1}{d}\ln\left(\frac{x}{d-x}\right)\right)\Bigg|_{x=a}^{d-a}\\ &= \frac{\mu I l}{2\pi}\left(\ln\left(\frac{d-a}{a}\right) - \ln\left(\frac{a}{d-a}\right)\right)\\ &= \frac{\mu I l}{2\pi}\times 2\ln\left(\frac{d-a}{a}\right) = \frac{\mu I l}{\pi}\ln\left(\frac{d-a}{a}\right).\end{aligned}$$

Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

Since the number of 'turns' in this structure is 1, the flux linkage is the same as magnetic flux: $\Lambda = \Phi$. the total inductance over the length l is given as:

$$L = \frac{\Lambda}{I} = \frac{\Phi}{I} = \frac{\mu l}{\pi} \ln\left(\frac{d-a}{a}\right) \quad \text{(H)}.$$

Therefore, the inductance per unit length is

$$L' = \frac{L}{l} = \frac{\mu}{\pi} \ln\left(\frac{d-a}{a}\right) \approx \frac{\mu}{\pi} \ln\left(\frac{d}{a}\right) \quad \text{(H/m)},$$

where the last approximation recognizes that the wires are thin compared to the separation distance (i.e., that $d \gg a$). This has been an implied condition from the beginning of this analysis, where the flux passing through the wires themselves have been ignored.

Problem-2: In terms of the d-c current I, how much magnetic energy is stored in the insulating medium of a 3-m-long, air-filled section of a coaxial transmission line, given that the radius of the inner conductor is 5 cm and the inner radius of the outer conductor is 10 cm?

Solution: the inductance per unit length of an air-filled coaxial cable is given by

$$L' = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \quad \text{(H/m)}.$$

Over a length of 2 m, the inductance is

$$L = 2L' = \frac{3 \times 4\pi \times 10^{-7}}{2\pi} \ln\left(\frac{10}{5}\right) = 416 \times 10^{-9} \quad \text{(H)}.$$

From $W_m = LI^2/2 = 208I^2$ (nJ), where W_m is in nanojoules when I is in amperes. Alternatively, we can use Eq. (5.106) to compute W_m:

$$W_m = \frac{1}{2} \int_{\mathcal{V}} \mu_0 H^2 \, d\mathcal{V}.$$

From Equation , $H = B/\mu_0 = I/2\pi r$, and

$$W_m = \frac{1}{2} \int_{z=0}^{3m} \int_{\phi=0}^{2\pi} \int_{r=a}^{b} \mu_0 \left(\frac{I}{2\pi r}\right)^2 r \, dr \, d\phi \, dz = 208I^2 \quad \text{(nJ)}.$$