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Single-phase Induction Motors

1 INTRODUCTION

Single-phase induction motors have numerous and diversified applications, both in homes and the industry. It is probably safe to say that single-phase induction motor applications far outweigh three-phase motor applications in the domestic sector. At homes, normally only single-phase power is provided, since power was originally generated and distributed to provide lighting. For this reason, early motor-driven appliances in homes depended on the development of the single-phase motor. Single-phase induction motors are usually small-sized motors of fractional kilowatt rating. They find wide applications in fans, washing machines, refrigerators, pumps, toys, hair dryers, etc. Single-phase induction motors operate at low power factors and are less efficient than three-phase induction motors.

2 PRODUCTION OF TORQUE

From the study of three-phase induction motors, it is seen that the three-phase distributed stator winding sets up a rotating magnetic field which is fairly constant in magnitude and rotates at synchronous speed. In a single-phase induction motor, there is only single-field winding excited with alternating current and, therefore, it is not inherently self-starting since it does not have a true revolving field. Various methods have been devised to initiate rotation of the squirrel-cage rotor and the particular method employed to start the rotor of single-phase motor will designate the specific type of motor.

Consider the behaviour of the magnetic field set up by an ac current in the single-phase winding. With reference to Fig 1, when the sinusoidal current is flowing in the field winding, neglecting the saturation effects of the

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magnetic iron circuit, the flux through the armature will vary sinusoidally with time. The magnetic field created at a particular instant of time will reverse during the next half-cycle of the ac supply voltage. Since the flux is pulsating, it will induce currents in the rotor bars which, in turn, will create a rotor flux which by Lenz's law opposes that of the main field. The direction of the rotor current as well as the torque created can also be determined. It is apparent that the clockwise torque produced is counteracted by the counter-clockwise torque and so no motion results, i.e. the motor is at standstill.

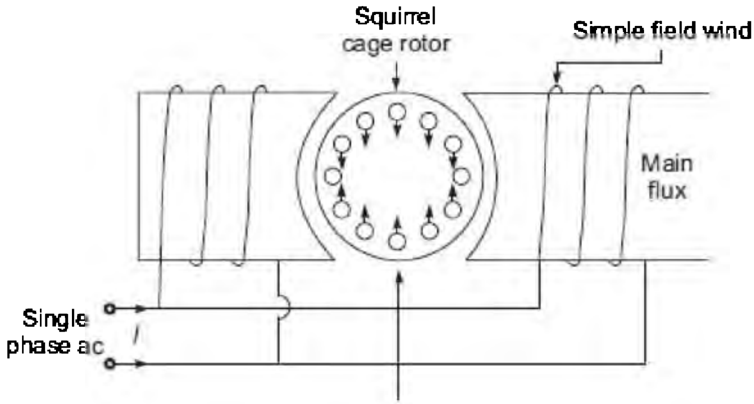

Fig. .1 Torque produced in the squirrel cage of a single-phase induction motor

Hence, a single-phase induction motor has no inherent self-starting torque.

When the rotor is made to rotate by auxiliary means in any one direction, it continues running in that direction. For explaining the running performance of the single-phase induction motor, two different theories, viz. revolving-field theory and cross-field theory have been adopted.

2.1 Revolving-Field Theory

According to this theory, the pulsating stationary mmf wave can be revolved into two counter-rotating mmf waves of equal magnitudes and rotating at synchronous speed. If two sinusoidally distributed mmfs, each of magnitude $F_{max}/2$, rotate in opposite directions, their combined effect is equivalent to one pulsating field $F_{max} \cos \omega t$ varying between $+F_{max}$ and $-F_{max}$ which is shown in Fig. (.2(a). A physical interpretation of the two oppositely rotating field components is depicted in Fig. (.2(b). Assuming the stator mmf wave to be sinusoidally distributed in space and varying sinusoidally with time, it can be represented as $F_S = F_S$ max sin ωt cos α where F_S max is the peak value corresponding to maximum instantaneous alternating current in stator winding and α is the space-displacement angle measured from the stator main-winding axis. Here, sin ωt indicates that mmf variation is sinusoidal with time and the term cos α indicates its co-sinusoidal distribution in space along the air-gap periphery.

Now
$$F_s = F_{s\,max} \sin \omega t \cos \alpha$$
$$= \frac{1}{2} F_{s\,max} \sin(\omega t - \alpha) + \frac{1}{2} F_{s\,max} \sin(\omega t + \alpha)$$

Hence, the two mmf components have maximum value of $\frac{1}{2} F_{s\,max}$ and they travel in opposite directions. The first wave whose argument is (ωt – α) travels in the forward direction and the other, whose argument is ($\omega t + \alpha$), travels in the backward direction. Each field component acts independently on the rotor in a similar fashion as the rotating field in a three-phase induction motor. The only difference is that here there are two fields, one tending to rotate the rotor clockwise and the other tending to rotate the rotor anticlockwise.

If the rotor runs at a speed n_r the rotor speed relative to the forward field is ($n_S – n_r$) where n_S is the synchronous speed.

∴ slip of the rotor current due to forward field

$$s_f = \frac{n_s + n_r}{n_s} = s$$

The rotor speed relative to the backward field is $n_S + n_r$.

∴ slip of the rotor current due to backward field is

$$s_b = \frac{n_s + n_r}{n_s} = 2 - s$$

Hence, the forward field induces rotor currents of frequency *sf* and backward field of frequency (2 – *s*) *f* in the same rotor conductors.

At standstill, both forward and backward fields rotate at synchronous speed

n_S with respect to rotor conductors. Hence, both these fields induce equal rotor emfs, equal rotor currents and produce equal rotor mmfs. These rotor mmfs react with their corresponding equal counter-rotating stator mmfs and hence both forward and backward rotating fields are equal with the rotor at standstill.

When the rotor rotates at a speed of n_r, the speed of forward flux wave is $(n_S - n_r)$ with respect to rotor conductors. This small relative speed induces small rotor emf, small rotor current which, in turn, produces small rotor mmf as compared to their values at standstill. The rotor frequency *sf* results in less rotor leakage reactance and hence a better rotor power factor. The reduced rotor mmf component at a better power factor is much smaller than its magnitude at standstill and it opposes the constant forward mmf wave. Hence, the forward flux wave becomes higher than its value at standstill. On the other hand, at rotor speed n_r, the speed of backward field becomes $n_S + n_r$ with respect to the rotor conductors. Due to this high relative speed, the backward field induces large rotor emf, large rotor current and hence large rotor mmf, as compared to their standstill values. As the rotor frequency is $(2 - s)f$, the rotor power factor is poor. This large rotor mmf component at poor power factor is much greater than its value at standstill which opposes the constant backward rotating mmf wave. Hence, the backward flux wave is much reduced from its magnitude at standstill.

From the above discussion, it is clear that as the speed increases, the backward flux wave decreases and the forward flux wave increases. Hence, in the under running condition, the forward field torque T_f is greater than the backward field torque T_b and the net torque $(T_f - T_b)$ is in the direction of rotor rotation. In the normal operating region, the torque speed curve of a single-phase induction motor is not far inferior from the three-phase induction motor as the forward flux wave at small slips is several times greater than the backward flux wave and the resultant air-gap field is similar to three-phase induction motor.

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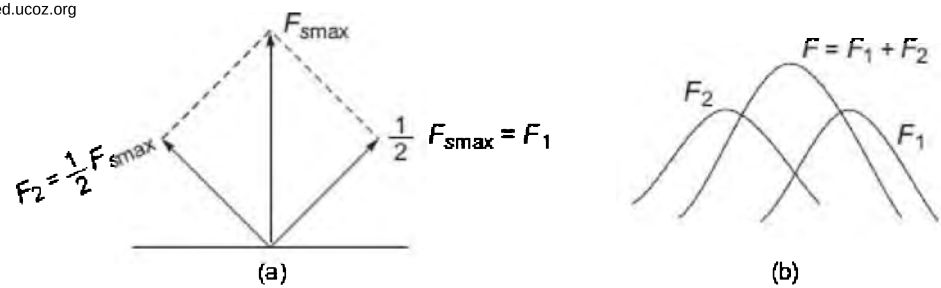

Fig. .2 Pulsating field resolved into two oppositely rotating fields

The torque–slip curve of the motor can be obtained by applying the principle of superposition. The superposition of two torques T_f and T_b, as shown in Fig. 3, gives the required torque–slip curve of a single-phase induction motor. From Fig. 3, it can be inferred that when the rotor speed is zero and the rotor speed is slightly less than n_s, the motor torque T is zero. Also, the motor can run in either direction.

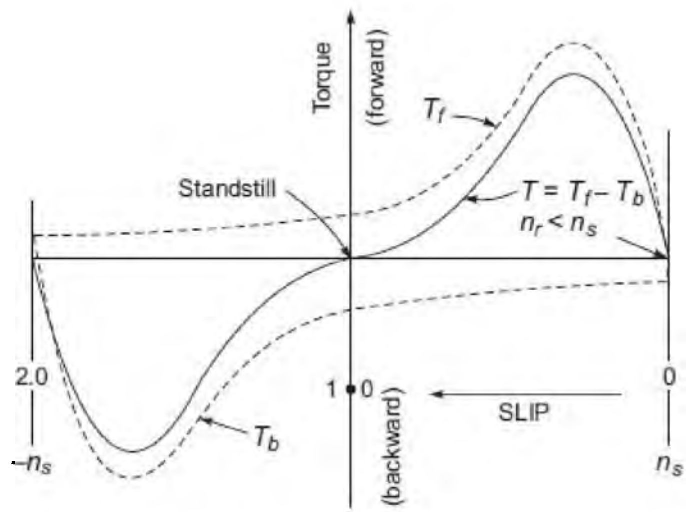

Fig. 3 Torque-speed characteristics of a single-phase induction motor

The single-phase induction motor having a simple winding, once started, will continue to run in the direction in which it is started. The manual self-starting is not a desirable practice and modifications are introduced to obtain

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the torque required to start. To accomplish this, a quadrature flux component in time and space with the stator flux must be provided at standstill. Auxiliary windings are normally placed on the stator to provide starting torque.

2.2 Cross-field Theory

The cross-field theory is an application of two axes or generalized theory of electrical machines. The performance of induction motor under normal running condition can be explained by this theory. Hence the flux is resolved into two components—one component acting along the stator winding axis and the other at right angles to it as shown in Fig. 4. x_1 x_2 and y_1 y_2 represent the two groups of short-circuited rotor conductors. x_1 x_2 links with one component flux and y_1 y_2 links with the other component flux.

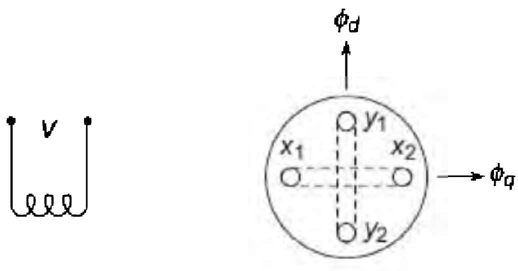

Fig. 4 Cross-field theory of single-phase induction motors

When the motor is at standstill, the stator applied voltage V sets up an mmf in q-axis and this mmf causes flux ϕ_q in this axis. This flux links with turn y_1 y_2 and induces the transformer emf in this turn which causes a current I_q. This current sets up an opposing mmf along the q-axis. Actually the resultant of these two mmfs sets up the ϕ_q. As both these mmfs are along the same axis, no torque is produced. The axis of the coil x_1x_2 is 90° to q-axis. Hence, no emf is induced in this coil and no current flows. Therefore, there is no torque in either coils.

When the motor is running, a back emf is induced in the stator winding which is equal to the voltage applied to the stator provided the voltage drops in the stator resistance and leakage reactances are neglected. The flux ϕ_q is constant. The flux ϕ_q sets up a motional emf in the coil x_1x_2. A current I_d starts flowing in the coil as it is short-circuited. This current sets up an mmf

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and flux ϕ_d in the *d*-axis.

The interaction of flux ϕ_q with current I_d and of flux ϕ_d with current I_q produces torque and the motor continues to run.

3 EQUIVALENT CIRCUIT

At standstill, the equivalent circuit of a single-phase induction motor is exactly similar to that of a transformer on short circuit. The equivalent circuit at standstill condition is shown in Fig. (5(a). R_c and X_φ represent the core loss and magnetizing reactance. R_1 and x_1 are the resistance and leakage reactance of the stator, $\left(\frac{r_2'}{2s}\right)$ and $\left(\frac{r_2'}{2(2-s)}\right)$ are the resistance and leakage reactance of the rotor referred to the stator.

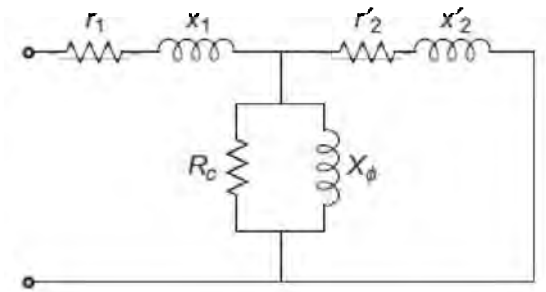

Fig. .5(a) Equivalent circuit of a single-phase induction motor at standstill

The air-gap flux can be resolved into two oppositely rotating components. These components at standstill are equal in magnitude, each one contributing an equal share to the resistive and reactive voltage drops in the rotor circuit. Hence, r_2 and x_2 can be split into two parts, each one corresponding to the effects of one of the magnetic fields. E_f and E_b are the voltages set up by the two oppositely rotating fields, viz. *forward* and *backward* rotating fields respectively. The equivalent circuit considering the effects of forward and backward flux components is shown in Fig. 5(b).

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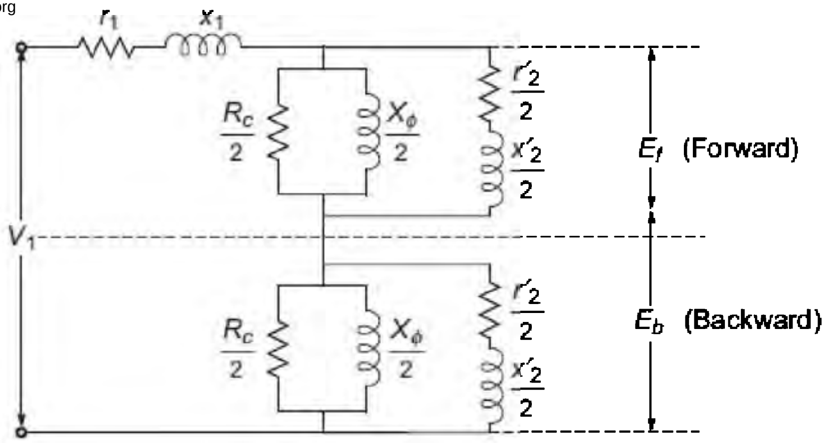

Fig. 5(b) Equivalent circuit at standstill showing the effects of forward and backward flux components

When the motor is running at a slip s, the slip for the forward field is s and for the backward field is $(2 - s)$. Hence, the resistance in the forward field becomes $\left(\frac{r_2'}{2s}\right)$ and in the backward field becomes $\left(\frac{r_2'}{2(2-s)}\right)$ As s is normally very small, $\left(\frac{r_2'}{2s}\right)$ is much higher than $\left(\frac{r_2'}{2(2-s)}\right)$ Hence, E_f is much greater than E_b.

Then equivalent circuit at any slip (s) is shown in Fig. 6.

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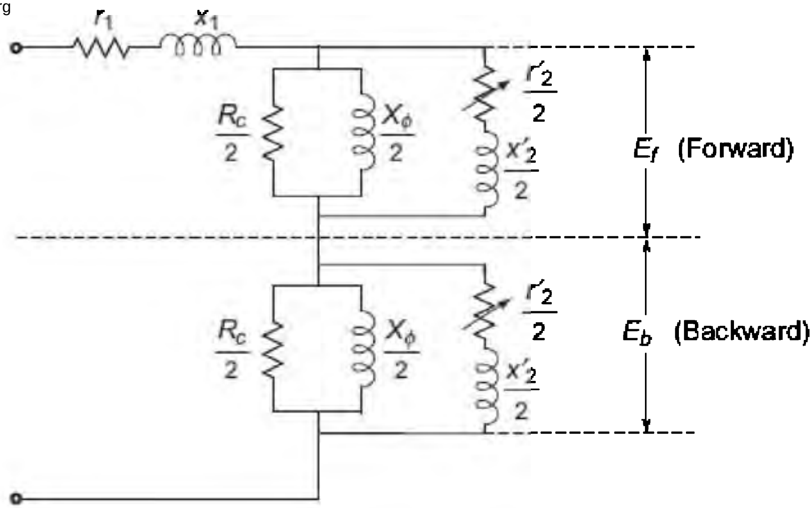

Fig. 6 Equivalent circuit of a single-phase induction motor at any slip

$$P_{gf} = \text{Air-gap power of forward field} = (I'_{2f})^2 \frac{r'_2}{2s} \text{ W}$$

$$P_{gb} = \text{Air-gap power of backward field} = (I'_{2b})^2 \frac{r'_2}{2(2-s)} \text{ W}$$

$$T_f = \text{Torque due to forward field} = \frac{P_{gf}}{2\pi n_s} \text{ Nm}$$

$$T_b = \text{Torque due to backward field} = \frac{P_{gb}}{2\pi n_s} \text{ Nm}$$

Net torque, $T = T_f - T_b$

Rotor copper loss due to forward field (P_{cu}(rot.)f) = sP_{gf}

Rotor copper loss due to backward field (P_{cu}(rot.) b) = $(2 - s)P_{gb}$

Total rotor copper loss (P_{cu}(rot.) = $sP_{gf} + (2 - s)P_{gb}$

Mechanical power developed $(=P_m) = (1 - s)\ (P_{gf} - P_{gb})$

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Problem 1

A 200 W, 240 V, 50 Hz single-phase induction motor runs on rated load with a slip of 0.05 p.u. The parameters are

$$r_1 = 11.4\,\Omega,\ x_1 = 14.5\,\Omega,$$
$$r_2' = 13.8\,\Omega,\ x_2' = 14.4\,\Omega,\ X_\phi = 270\,\Omega$$

Calculate (a) power factor, (b) input power, and (c) efficiency.

Solution

From Fig. 6, neglecting the core loss resistance R_C, the total series impedance

$$Z = r_1 + jx_1 + \frac{j\dfrac{X_\phi}{2}\left(\dfrac{r_2'}{2s} + j\dfrac{x_2'}{2}\right)}{\dfrac{r_2'}{2s} + j\left(\dfrac{X_\phi}{2} + \dfrac{x_2'}{2}\right)} + \frac{j\dfrac{X_\phi}{2}\left(\dfrac{r_2'}{2(2-s)} + j\dfrac{x_2'}{2}\right)}{\dfrac{r_2'}{2(2-s)} + j\left(\dfrac{X_\phi}{2} + \dfrac{x_2'}{2}\right)}$$

$$= 11.4 + j14.5 + \frac{-\dfrac{270\times 14.4}{4} + j\dfrac{270\times 13.8}{4\times 0.05}}{\dfrac{13.8}{2\times 0.05} + j\left(\dfrac{270}{2} + \dfrac{14.4}{2}\right)} + \frac{-\dfrac{270\times 14.4}{4} + j\dfrac{270\times 13.8}{4(2-0.05)}}{\dfrac{13.8}{2(2-0.05)} + j\dfrac{270+14.4}{2}}$$

$$= 11.4 + j14.5 + \frac{-972 + j18630}{138 + j142.2} + \frac{-972 + j477.69}{3.538 + j142.2}$$

$$= 11.4 + j14.5 + \frac{18655\angle 92.98^\circ}{198\angle 45.86^\circ} + \frac{1083\angle 153.83^\circ}{142.24\angle 88.57^\circ}$$

$$= 11.4 + j14.5 + 94.22\angle 47.12^\circ + 7.6\angle 65.25^\circ$$
$$= (11.4 + 64.11 + 3.18) + j(14.5 + 69 + 6.9)$$
$$= (78.69 + j90.4)\ \Omega$$

$$\therefore \text{input current} = \frac{240\angle 0^\circ}{78.69 + j90.4} = \frac{240\angle 0^\circ}{119.85\angle 48.96^\circ} = 2\angle -48.96^\circ\ \text{A}$$

Hence, power factor is (cos 48.96°) lagging, i.e. 0.656 lagging.
Input power = 240 × 2 × 0.656 W, i.e. 314.88 W.

Output power is 200 W.

Hence, efficiency = $= \frac{\text{Output}}{\text{Input}} = \frac{200}{314.88} = 0.638$, i.e. 63.5%

Problem 2

A 230 V, 50 Hz. 4-pole single-phase induction motor has the following parameters:

$$r_1 = 2.51\ \Omega,\ x_1 = 4.62\ \Omega,\ r_2' = 7.81\ \Omega,\ x_2' = 4.62\ \Omega$$

and $X_\phi = 150.88\ \Omega$

Determine the stator main winding current and power factor when the motor is running at a slip of 0.05.

Solution

The total series impedance is obtained as

$$Z = 2.51 + j4.62 + \frac{j\frac{150.88}{2}\left(\frac{7.81}{2\times 0.05} + j\frac{4.62}{2}\right)}{\frac{7.81}{2\times 0.05} + j\left(\frac{4.62}{2} + \frac{150.88}{2}\right)} + \frac{j\frac{150.88}{2}\left\{\frac{7.81}{2(2-0.05)} + j\frac{4.62}{2}\right\}}{\frac{7.81}{2(2-0.05)} + j\left(\frac{150.88}{2} + \frac{4.62}{2}\right)}$$

$$= 2.51 + j4.62 + \frac{-174.26 + j5891.86}{78.1 + j77.75} + \frac{174.26 + j151.07}{2 + j77.75}$$

$$= 2.51 + j4.62 + \frac{5894.40 \angle 91.69^\circ}{110.20 \angle 44.87^\circ} + \frac{230.62 \angle 139.077^\circ}{77.77 \angle 88.50}$$

$$= 2.51 + j4.62 + 53.48 \angle 46.82^\circ + 2.965 \angle 50.58^\circ$$

$$= (2.51 + 36.596 + 1.88) + j(4.62 + 39 + 2.3)$$

$$= 40.986 + j45.92 = 61.54 \angle 48.25^\circ\ \Omega$$

Stator main winding current is $\frac{230 \angle 0^\circ}{61.54 \angle 48.25^\circ}$, i.e. $3.73 \angle -48.25^\circ$ A

Hence, power factor is (cos 48.25°) i.e. 0.666 lagging.

Problem 3

In a 6-pole, single-phase induction motor, the gross power absorbed by the forward and backward fields are 160 W and 20 W respectively. If the motor speed is 950 rpm

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and the no-load frictional loss is 75 W, find the shaft torque.

Solution

Air-gap power of forward field P_{gf} = 160 W

Air-gap power of backward field P_{gb} = 20 W

Net power = $P_{gf} - P_{gb}$ = 160 W – 20 W = 140 W

$$\text{Synchronous speed } N_s = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Speed of motor N_r = 950 rpm

$$\text{Hence, slip } s = \frac{1000 - 950}{1000} = 0.05$$

Power output is (1 – *s*) × 140 – 75 = 58 W (= shaft power)

$$\text{Shaft torque} = \frac{\text{shaft power}}{2\pi \times \frac{950}{60}} = \frac{58}{2\pi \times \frac{95}{6}} = 0.58 \text{ Nm.}$$

.4 DETERMINATION OF PARAMETERS OF EQUIVALENT CIRCUIT

The parameters of the equivalent circuit of a single-phase induction motor can be determined from the *no-load* and *blocked rotor* test.

.4.1 Blocked-Rotor Test

In this test, a very small voltage is applied to the stator and the rotor is blocked (care is to be taken such that the stator current does not exceed the full-load current.)

The voltage, current and power input to the stator are measured. When the rotor is blocked, *s* = 1 and hence parallel combination $\left(\frac{R_c}{2}\right)$ and $\left(\frac{X_\phi}{2}\right)$ is much greater than $\left[\frac{r_2'}{2} + j\frac{x_2'}{2}\right]$ (in Fig. 6). Therefore, under blocked rotor test, the equivalent circuit reduces to that shown in Fig. 7. Since (R_c/2) and (X_ϕ/2) are of very high values, hence they can be neglected in the equivalent circuit.

Let V_{SC}, I_{SC} and W_{SC} be the input voltage, current and power during blocked rotor test.

The total resistance $$(r_1 + r'_2) = \frac{W_{sc}}{I_{sc}^2} = R_{sc}$$

Total impedance, $$Z_{sc} = \frac{V_{sc}}{I_{sc}}$$

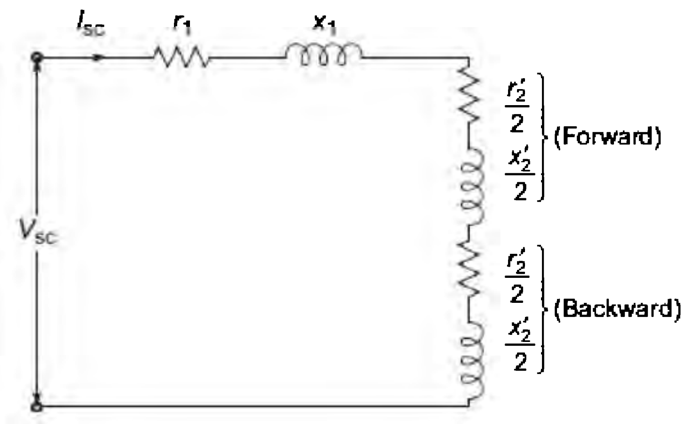

Fig. 7 Equivalent circuit under blocked rotor condition

Hence, total reactance $(x_1 + x'_2) = \sqrt{Z_{sc}^2 - R_{sc}^2}$

Generally, $r_1 = r'_2$ and $x_1 = x'_2$. Hence, r_1, r_1, r'_2, x_1, x_1 and x'_2 can be determined from this test.

4.2 No-Load Test

In this test, the motor is run on no-load condition and voltage V_o, current I_o and power W_o of the stator are measured. At no load, s is very small and core loss resistance R_c is neglected. Hence,

from Fig. 6, $\left(\frac{r'_2}{s}\right)$ is much greater than $\left(\frac{X_\phi}{2}\right)$. Also, $\frac{r'_2}{2(2-s)}\left(=\frac{r'_2}{4}\right)$ is much smaller than $X\phi/2$. Therefore, under no-load condition, the equivalent circuit can be reduced to that shown in Fig. 8.

Here, $\left(\frac{r_2'}{s}\right)$ and $\left(\frac{X_\phi}{2}\right)$ are neglected in equivalent circuit.

No load p.f. $\frac{W_o}{V_o I_o}$

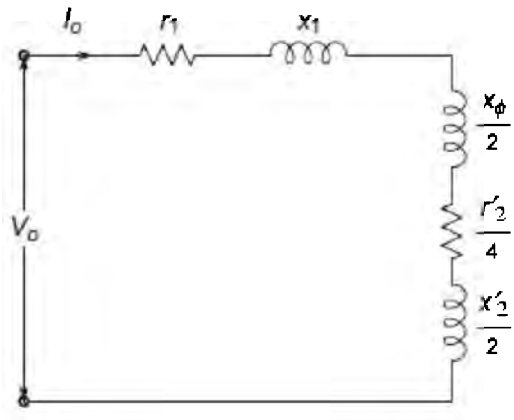

Fig. 8 Equivalent circuit under no-load condition

Now voltage across $\left(\frac{X_\phi}{2}\right)$ is $\left[V_o - I_o \angle \phi_o \left\{\left(r_1 + \frac{r_2}{4}\right) + j\left(x_1 + \frac{x_2'}{2}\right)\right\}\right]$

Hence, $$\frac{X_\phi}{2} = \frac{V_o - I_o \angle -\phi_o \left[\left(r_1 + \frac{r_2'}{4}\right) + j\left(x_1 + \frac{x_2'}{2}\right)\right]}{I_o}$$

and X_ϕ can thus be determined.

Problem 4

The main winding of an 8-pole, single-phase induction motor is excited from 240 V, 50 Hz supply and the motor takes a line current of $50\angle{-50^\circ}$ A at standstill. Determine the torque at a slip of 4% neglecting stator impedance, magnetizing current rotational loss.

Solution

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From Fig. 5(a), neglecting the stator impedance and magnetizing reactance

$$Z_2 = r_2' + jx_2' = \frac{240}{50\angle -50^\circ} = 4.8\angle 50^\circ\ \Omega = 3.08 + j3.68$$

$\therefore$ $$r_2' = 3.08\ \Omega \text{ and } x_2' = 3.68$$

From Fig. 6, at slip $s = 0.04$,

Forward impedance $$Z_f = R_f + jX_f = \frac{r_2'}{2s} + j\frac{x_2'}{2}$$

$$= \frac{3.08}{2\times 0.04} + j\frac{3.68}{2}$$

$$= 38.5 + j1.84\ \Omega$$

Backward impedance $$Z_b = R_b + j\times b = \frac{r_2'}{2(2-s)} + j\frac{x_2'}{2}$$

$$= \frac{3.08}{2(2-0.04)} + j\frac{3.68}{2}$$

$$= 0.786 + j\,1.84\ \Omega$$

$\therefore$ total input impedance when the motor is running at a slip of 4% is

$$Z_1 = R_f + jX_f + R_b + jX_b$$
$$= 39.286 + j\,3.68\ \Omega \quad \text{(neglecting stator impedance)}$$

The stator current $$I_1 = \frac{V}{Z_1} = \frac{240\angle 0^\circ}{39.286 + j3.68}\ \text{A}$$

$$= \frac{240\angle 0^\circ}{39.457\angle 5.35^\circ}\ \text{A}$$

$$= 6.08\angle -5.35^\circ\ \text{A}$$

Air-gap power of forward field

$$P_{gf} = I_1^2\frac{r'_2}{2s} = I_1^2 R_f$$

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Air-gap power of backward field

$$P_{gb} = I_1^2 \frac{r'_2}{2(2-s)} = I_1^2 R_b$$

∴ mechanical power developed

$$P_m = (1 - s)\,(P_{gf} - P_{gb}) = (1 - 0.04) \times (6.08)^2 \times (R_f - R_b)$$

$$= 35.488\,(38.5 - 0.786)$$
$$= 1338.39 \text{ W}$$

Torque developed $$T = \frac{P_m}{\omega_r} = \frac{P_m}{(1-s)\omega_s}$$

$$= \frac{1338.39}{(1-0.04) \times 2\pi \times \frac{2 \times 50}{8}} \text{ Nm}$$

$$= 17.76 \text{ Nm.}$$

Problem 5

The data obtained from tests of a 230 V, 300 W, 50 Hz, 8-pole single induction motor are as follows:

No-load test:	*230 V,*	*140 W,*	*3 A*
Blocked-rotor test:	*115 V,*	*400 W,*	*6 A*

If the stator winding resistance is 3 Ω, determine the equivalent circuit parameters if the starting winding is open during blocked rotor test. Also, determine the core, friction and windage losses.

Solution

From blocked-rotor test,

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$$Z_{sc} = \frac{V_{sc}}{I_{sc}} = \frac{115}{6} = 19.17\ \Omega$$

$$R_{sc} = r_1 + 2\left(\frac{r_2'}{2}\right) = \frac{400}{(6)^2}\ \Omega = 11.11\ \Omega$$

∴ $$X_{sc} = x_1 + 2\left(\frac{x_2'}{2}\right) = \sqrt{(19.17)^2 - (11.11)^2}$$

$$= 15.62\ \Omega$$

Now, $$r_1 = 3\ \Omega$$

∴ $$r_2' = 11.11 - 3 = 8.11\ \Omega$$

and $$x_1 = x_2' = \frac{1}{2} X_{sc} = 7.81\ \Omega$$

From no-load test,

$$Z_{nl} = \frac{230}{3}\ \Omega = 76.67\ \Omega$$

No-load power factor $$\cos\ \theta_{nl} = \frac{140}{230 \times 3} = 0.2$$

∴ $$\sin\ \theta_{nl} = 0.98$$

∴ $$X_{nl} = x_1 + \frac{x_\phi}{2} + \frac{x_2'}{2} = Z_{nl} \sin\ \theta = 76.67 \times 0.98$$

$$= 75.1366\ \Omega$$

Now $x_1 = x_2' = 7.81\ \Omega$

∴ $$X_\phi = 2\left(75.1366 - 7.81 - \frac{7.81}{2}\right)$$

$$= 126.84\ \Omega$$

The power input to a single-phase induction motor at no load is equal to core, friction, windage and ohmic loss.

From Fig. .8, the ohmic loss at no load is

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$$I_{nl}^2\left(r_1+\frac{r_2'}{4}\right)=3^2\left(3+\frac{8.11}{4}\right)=45.2475\ \text{W}$$

∴ the core, friction and windage losses are

$$140-45.2475=94.7525\ \text{W}$$

5 STARTING OF SINGLE-PHASE INDUCTION MOTORS

Since a single-phase induction motor does not have a starting torque, it needs special methods of starting. The stator is provided with two windings, called *main* and *auxiliary windings,* whose axes are space displaced by 90 electrical degrees. The auxiliary winding is excited by a current which is out of phase with the current in the main winding, both currents derived from the same supply. If the phase difference between the two currents is 90° and the mmfs created by them are equal, maximum starting torque is produced. If the phase difference is not 90° and the mmfs are equal, the starting torque will be small, but in many applications, it is still sufficient to start the motor. The auxiliary winding may be disconnected by a centrifugal switch after the motor has achieved about 75% speed.

Single-phase induction motors are usually classified according to the auxiliary means used to start the motors. They are classified as follows:

1. Split-phase motor
2. Capacitor-start motor
3. Capacitor-start capacitor-run motor
4. Shaded-pole motor

6 SPLIT-PHASE INDUCTION MOTORS

One of the most widely used types of single-phase motors is the *split-phase* induction motor. Its service includes a wide variety of applications such as refrigerators, washing machines, portable hoists, small machine tools, blowers, fans, centrifugal pumps, etc.

The essential parts of the split-phase motor is shown in Fig. 9(a). It shows auxiliary winding, also called the *starting winding,* in space quadrature, i.e., 90 electrical degrees displacement with the main stator winding. The rotor is normally of squirrel-cage type. The two stator windings are connected in parallel to the ac supply. A phase displacement between the winding currents is obtained by adjusting the winding impedances, either by inserting a resistor in series with the starting winding or as is generally the practice, by using a smaller gauge wire for the starting winding. A phase displacement of 30° between the currents of main winding I_m and auxiliary winding I_a can be achieved at the instant of starting. A typical phasor diagram is shown in Fig. 9(b).

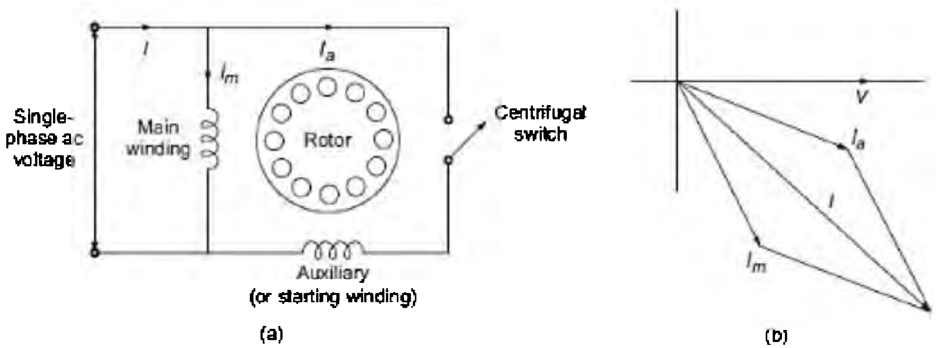

Fig. 9 Split phase motor: (a) Schematic representation (b) Phasor diagram

When the motor has come to about 70 to 75% of synchronous speed, the starting winding may be opened by a centrifugal switch and the motor will continue to operate as a single-phase motor. At the point where the starting winding is disconnected, the motor develops nearly as much torque with the main winding alone as it was with both windings connected. It can be observed from the typical torque-speed characteristic for this type of motor in Fig. 10. The starting winding is designed to take the minimum starting current from the required torque. The locked rotor starting current may be typically in the range 5 to 7 times the rated current while the starting torque is also about 1.5 to 2 times the rated torque. The high starting current is not objectionable since once started, it drops off almost instantly. The major disadvantages of this type of induction motor are relatively low starting torque and high slip. Moreover, the reversal of rotation can be made only when the motor is standstill (by reversing the line connections of either the main winding or the starting winding) but not while running. Also, the efficiency is lower.

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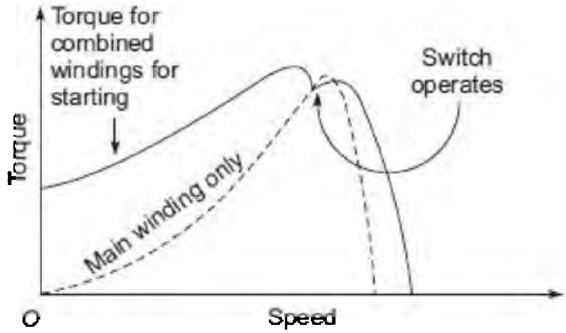

Fig. 10 Typical torque-speed characteristic of a general purpose split-phase motor

.7 CAPACITOR-START MOTOR

In the split-phase motor, the phase shift between the stator currents was accomplished by adjusting the impedances of the windings, i.e. by making the starting winding of a relatively higher resistance. This resulted in a phase shift of nearly 30°. Since the developed torque of any split-phase motor is proportional to the pole flux produced and the rotor current, it is also dependent on the angle between the winding currents. This implies that if a capacitor is connected in series with the starting winding, the starting torque will increase. By proper selection of the capacitor, the current in the starting winding will lead the voltage across it and a greater displacement between winding currents is obtained.

Figure .11 shows the capacitor-start motor and its corresponding phasor diagram indicating a typical displacement between winding currents of about 80°–90°. The value of the capacitor needed to accomplish this is typically 135 pF or a 1/4 h.p. motor and 175 pF for a 1/3 h.p. motor. Contrary to the split-phase motor discussed earlier, the speed of the capacitor-start motor under running conditions is reversible. If temporarily disconnected from the supply line, its speed will drop allowing the centrifugal switch to close. The connections to the starting winding are reversed during this interval and the motor is reconnected to the supply with closed centrifugal switch. The resulting rotating field will now rotate opposite to the direction in which the motor rotates. Since the current displacement between the windings is much larger in this motor compared to the split-phase motor, the torque being proportional to this will also be much larger and exceed the torque produced by the rotor. Therefore, the motor will slow down, stop and reverse its

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direction of rotation. When the speed reaches to about 75 to 80% of synchronous speed, the centrifugal switch opens and the motor will reach speed as dictated by the load.

Because of higher starting torques, capacitor-start motors are used in applications where not only higher starting torques are required but also where reversible motors are needed. Applications of capacitor motors are in washing machines, betted fans and blowers, dryers, pumps and compressors.

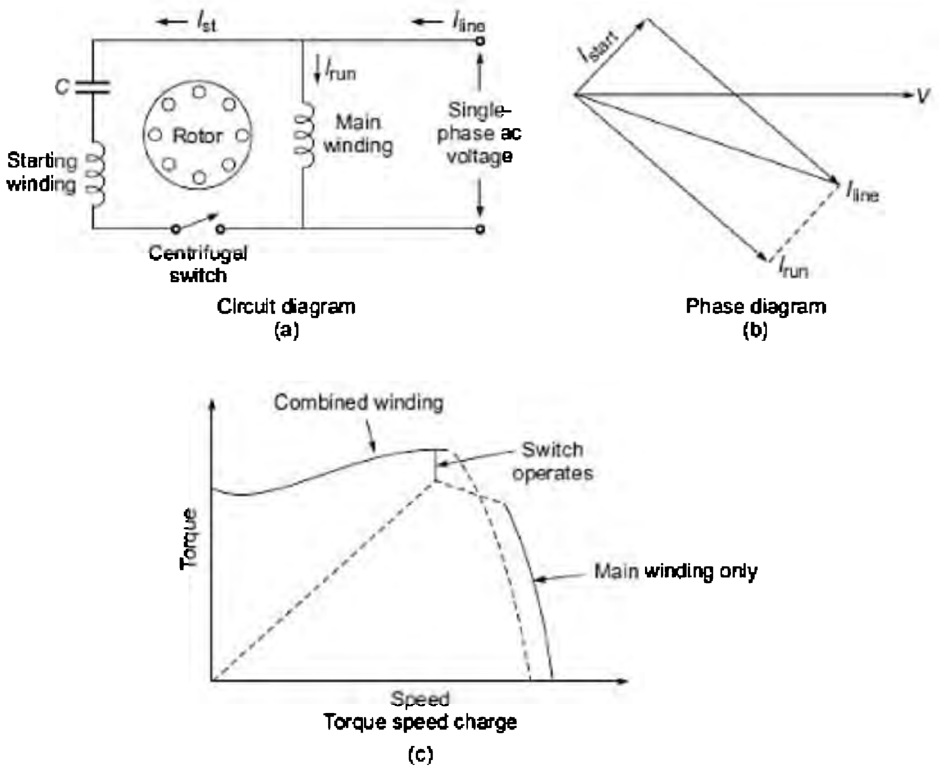

Fig. 11 Capacitor-start induction motor

8 CAPACITOR-STARTCAPACITOR-RUN MOTOR

The capacitor-start motor discussed above has still relatively low starting torque, although it is considerably better than the split-phase motor.

In case where higher starting torques are required, best results will be

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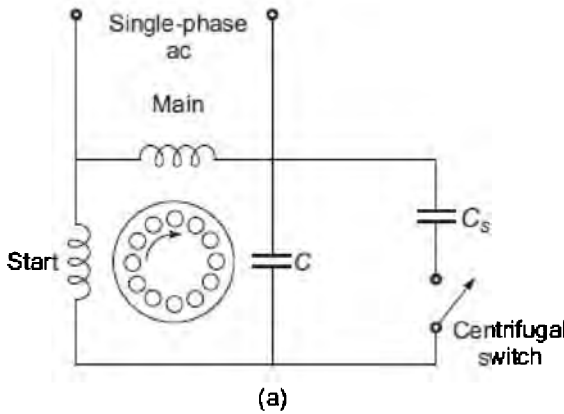
obtained if a large value of capacitance is used at start which is then gradually decreased as the speed increases. In practice, two capacitors are used for starting and one is cut out of the circuit by a centrifugal switch once a certain speed is reached, usually about 75% of full-load speed. This starting or intermittent capacitor is of fairly high capacity (usually of the order of 10 to 15 times the value of the running capacitor, which remains in the circuit.) Figure .12 illustrates the connection diagrams for the capacitor motor showing two methods generally encountered.

The first method shown in Fig. 12(a) uses an electrolytic capacitor in the starting circuit whose leakage is too high. The second capacitor is oil-filled which remains in the circuit always and has little leakage; it is therefore suitable for continuous operation.

The second circuit [Fig. 12(b)] uses an auto-transformer and one-oil filled high-voltage capacitor. This method utilizes the transformer principle of reflected impedance from the secondary to the primary. For instance, an auto-transformer with 180 turns tapped at the 30-turn point would reflect an 8 μF running capacitor to the primary as $\left(\frac{180}{30}\right)^2 \times 8\ \mu F = 288\ \mu F$, representing an increase of about 36 times. Thus, running an oil-filled capacitor may be used for starting purposes as well, by eliminating one capacitor in lieu of the auto-transformer, which is of comparable cost. Care must be taken to ensure that the capacitor can withstand the stepped-up voltage which is 180/30 = 6 times the rated voltage at start.

Like the capacitor-start motor, the capacitor-run motor may be damaged if the centrifugal switch fails to operate properly. The primary advantage of a capacitor-run motor or a two-value capacitor motor is its high starting torque, good running torque and quiet operation. Reversing the line leads to one of the windings in the usual manner causing motor operation in the opposite direction. It is, therefore, classified as a *reversible-type motor.* These motors are manufactured in a number of sizes from 1/8 to 3/4 hp and are used in compressors, conveyors, pumps and other high-torque loads.



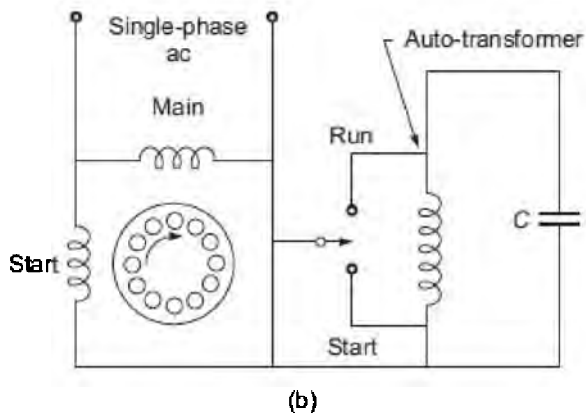


Fig. 12 Capacitor-start capacitor-run motor

9 SHADED-POLE MOTORS

Like any other induction motor, the *shaded-pole* motor is caused to run by the action of the magnetic field set up by the stator windings. There is, however, one extremely important difference between the polyphase induction motor and the single-phase induction motor discussed so far. As discussed, these motors have a truly rotating magnetic field, either circular, as in the three-phase machine, or of elliptical shape as encountered in most of the single-phase motors. In the shaded-pole motor, the field merely shifts from one side of the pole to the other. In other words, it does not have a rotating field but one that sweeps across the pole faces.

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An elementary understanding of how the magnetic field is created may be gained from the simple circuit in Fig. 9.13, illustrating the shaded-pole motor. As can be seen, the poles are divided into two parts, one of which is “shaded”, i.e., around the smaller of the two areas formed by a slot cut across the laminations, a heavy copper short circuited ring, called the *shading coil,* is placed. That part of the iron around which the *shading coil* is placed is called the shaded part of the pole. When the excitation winding is connected to an ac source, the magnetic field will sweep across the pole face from the unshaded to the shaded portion. This, in effect, is equivalent to an actual physical motion of the pole, the result is that the squirrel-cage rotor will rotate in the same direction.

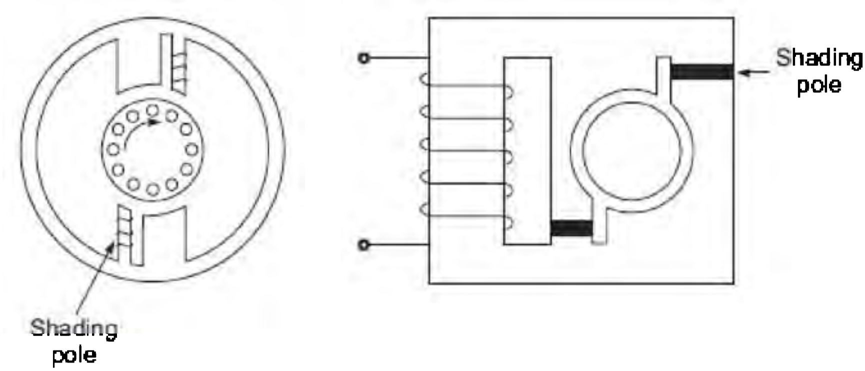

Fig. ' 13 Shaded-pole motor

To understand how this sweeping action of the field across the pole face occurs, let us consider the instant of time when the current flowing in the excitation winding is starting to increase positively from zero, as illustrated in Fig. . .14(a). In the unshaded part of the pole, the flux will start to build up in phase with the current. Similarly, the flux ϕ, in the shaded portion of the pole, will build up, but this flux change will induce a voltage in the shading coil which will cause current to flow. By Lenz’s law, this current flows in such a direction as to oppose the flux change that induces it. Thus, the building up of flux ϕ, in the shaded portion is delayed. It has the overall effect of shifting the axis of the resultant magnetic flux into the unshaded portion of the pole. When the current in the excitation coil is at or near the maximum value as indicated in Fig. 14(b), the flux does not change appreciably. With an almost constant flux, no voltage is induced in the shading coil and, therefore, it, in turn, does not influence the total flux. The result is that the resulting magnetic flux shifts to the centre of the pole.

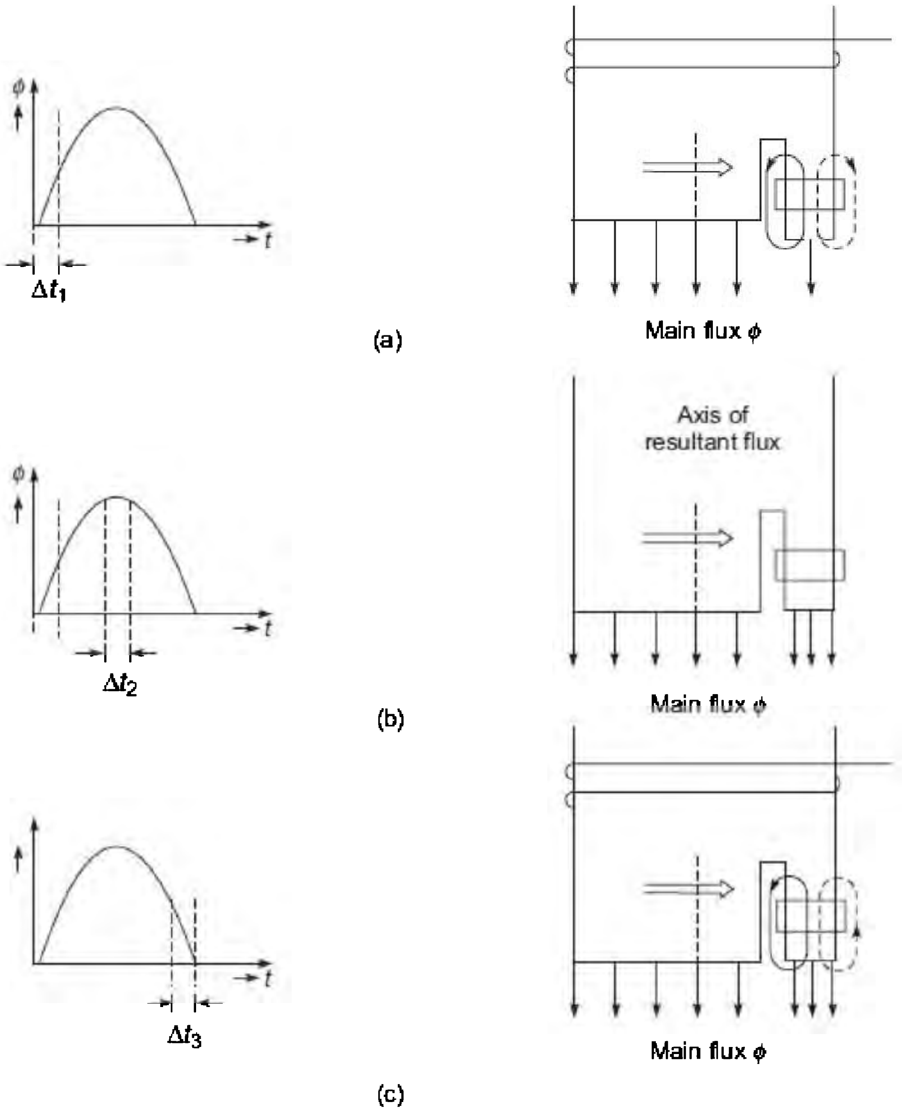

Fig. 14 Sweeping action of field across the pole

Figure 14(c) shows the current in the excitation coil decreasing. The flux in the unshaded portion of the pole decreases immediately. However, currents induced in the shading coil tend to oppose this decrease in flux; consequently, they try to maintain the flux. The result of this action translates into a movement of the magnetic flux axis towards the centre of the shaded portion of the pole. Hence, flux ϕ continues to lag behind the flux axis during this part of the cycle.

It can similarly be reasoned that at any instant of the current cycle, the flux ϕ lags behind in time. The net effect of this time and space displacement is to produce a gliding flux across the pole face and consequently in the air gap, which is always directed towards the shaded part of the pole. Therefore, *the direction of rotation of the shaded-pole motor is always from the unshaded towards the shaded part of the pole.*

Simple motors of this type cannot be reversed but must be assembled so that the rotor shaft extends from the correct end in order to drive the load in the proper direction. There are specially designed shaded-pole motors which are reversible. One form of design is to use two main windings and a shading coil. For one direction of rotation, one main winding is used and for the opposite rotation the other; such an arrangement is adaptable only to distributed windings, hence this necessitates a slotted stator.

Another method employed is to use two sets of open-circuited shading coils, one set placed on each side of the pole. A switch is provided to short circuit the shading coil. Depending on the rotational direction desired, offsetting the simple construction and a low cost of this motor. This motor has a low starting torque, little overload capacity and low efficiencies (5 to 35%).

These motors are built in sizes ranging from 1/250 hp up to about 1/20 hp. Typical applications of shaded-pole motors are where efficiencies are of minor concern such as in toys and fans.

Problem 9.9

A 200 V, 50 Hz capacitor-start motor has the following impedances at standstill:

Main winding Z_m = (8+j3) Ω

Auxiliarywinding Z_a = *(10+j8) Ω*

Find the value of capacitance to be connected in series with auxiliary winding to give phase displacement of 90° between currents in the two windags.

Solution

The phasor diagram is shown in Fig. 9.11(b).

Phase angle of current in main winding is $\left(\tan^{-1}\frac{3}{8}\right) = 20.55°$

With capacitor C in the auxiliary winding, the phase angle of current in auxiliary winding is $\left(\tan^{-1}\frac{8-\frac{1}{\omega C}}{10}\right)$

To give a phase displacement of 90° between the two winding currents, we can write,

$$-\tan^{-1}\frac{8-\frac{1}{\omega C}}{10} - (-20.55°) = 90°$$

i.e.

$$-\tan^{-1}\frac{8-\frac{1}{\omega C}}{10} = 69.45°$$

or

$$8-\frac{1}{\omega C} = 10\tan(-69.45°) = -26.67$$

Hence

$$\frac{1}{\omega C} = 34.67\ \Omega$$

or

$$C = \frac{1}{2\pi\times 50\times 34.67}\ \text{F} = 91.84\ \mu\text{F}.$$