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Problem 1. The rectangular loop shown in Fig. 1. consists of 20 closely wrapped turns and is hinged along the z-axis. The plane of the loop makes an angle of 30° with the y-axis, and the current in the windings is 0.5 A. What is the magnitude of the torque exerted on the loop in the presence of a uniform f eld $\mathbf{B} = \hat{\mathbf{y}}2.4$ T? When viewed from above, is the expected direction of rotation clockwise or counterclockwise?

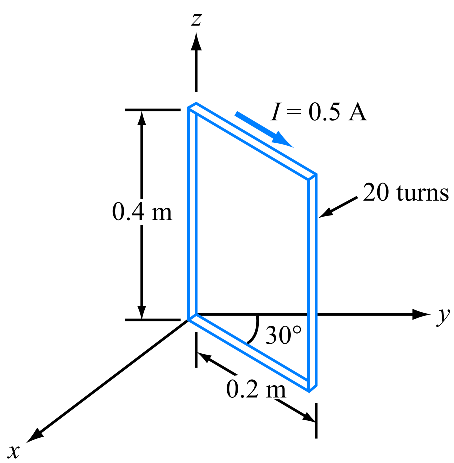

Fig 1: Hinged rectangular loop of Problem

Solution: The magnetic torque on a loop is given by $\mathbf{T} = \mathbf{m} \times \mathbf{B}$, where $\mathbf{m} = \hat{\mathbf{n}}NIA$. For this problem, it is given that $I = 0.5$ A, $N = 20$ turns, and $A = 0.2 \text{ m} \times 0.4 \text{ m} = 0.08 \text{ m}^2$. From the fig 1, $\hat{\mathbf{n}} = -\hat{\mathbf{x}}\cos 30° + \hat{\mathbf{y}}\sin 30°$. Therefore, $\mathbf{m} = \hat{\mathbf{n}}0.8 \text{ (A} \cdot \text{m}^2)$ and $\mathbf{T} = \hat{\mathbf{n}}0.8 \text{ (A} \cdot \text{m}^2) \times \hat{\mathbf{y}}2.4 \text{ T} = -\hat{\mathbf{z}}1.66$ (N·m). As the torque is negative, the direction of rotation is clockwise, looking from above.

Problem 2. In a cylindrical coordinate system, a 2-m-long straight wire carrying a current of 5 A in the positive z-direction is located at $r = 4$ cm, $\phi = \pi/2$, and $-1 \text{ m} \leq z \leq 1$ m.

(a) If $\mathbf{B} = \hat{\mathbf{r}}\,0.2\cos\phi$ (T), what is the magnetic force acting on the wire?

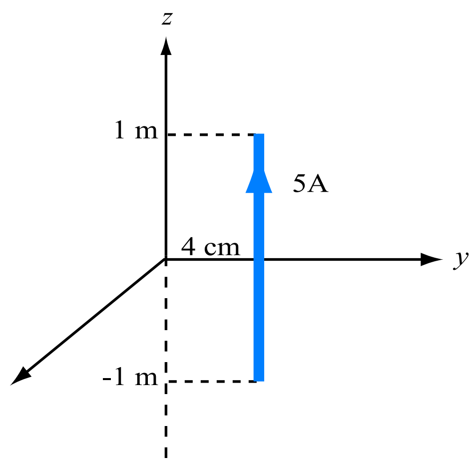

Fig 2.

(b) How much work is required to rotate the wire once about the z-axis in the negative ϕ-direction (while maintaining $r = 4$ cm)?

(c) At what angle ϕ is the force a maximum?

Solution:

(a)

$$\begin{aligned}\mathbf{F} &= I\boldsymbol{\ell} \times \mathbf{B} \\ &= 5\hat{\mathbf{z}}2 \times [\hat{\mathbf{r}}\,0.2\cos\phi] \\ &= \hat{\boldsymbol{\phi}}\,2\cos\phi.\end{aligned}$$

At $\phi = \pi/2$, $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}$. Hence,

$$\mathbf{F} = -\hat{\mathbf{x}}2\cos(\pi/2) = 0.$$

(b)

$$\begin{aligned}W &= \int_{\phi=0}^{2\pi} \mathbf{F}\cdot d\mathbf{l} = \int_0^{2\pi} \hat{\boldsymbol{\phi}}\,[2\cos\phi]\cdot(-\hat{\boldsymbol{\phi}})r\,d\phi\Big|_{r=4\text{ cm}} \\ &= -2r\int_0^{2\pi}\cos\phi\,d\phi\Big|_{r=4\text{ cm}} = -8\times10^{-2}\,[\sin\phi]_0^{2\pi} = 0.\end{aligned}$$

The force is in the $+\hat{\boldsymbol{\phi}}$-direction, which means that rotating it in the $-\hat{\boldsymbol{\phi}}$-direction would require work. However, the force varies as $\cos\phi$, which means it is positive

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when $-\pi/2 \leq \phi \leq \pi/2$ and negative over the second half of the circle. Thus, work is provided by the force between $\phi = \pi/2$ and $\phi = -\pi/2$ (when rotated in the $-\hat{\boldsymbol{\phi}}$-direction), and work is supplied for the second half of the rotation, resulting in a net work of zero.

(c) The force **F** is maximum when $\cos\phi = 1$, or $\phi = 0$.

Problem 1. A 20-turn rectangular coil with side $l = 20$ cm and $w = 10$ cm is placed in the y–z plane as shown in Fig.3.

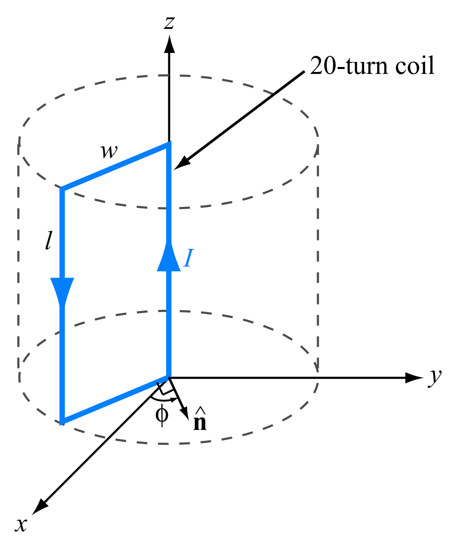

Fig 3: Rectangular loop

(a) If the coil, which carries a current $I = 10$ A, is in the presence of a magnetic f ux density

$$\mathbf{B} = 2 \times 10^{-2}(\hat{\mathbf{x}} + \hat{\mathbf{y}}2) \quad \text{(T)},$$

determine the torque acting on the coil.

(b) At what angle ϕ is the torque zero?

(c) At what angle ϕ is the torque maximum? Determine its value.

Solution:

(a) The magnetic f eld is in direction $(\hat{\mathbf{x}} + \hat{\mathbf{y}}2)$, which makes an angle $\phi_0 = \tan^{-1}\frac{2}{1} = 63.43°$.

The magnetic moment of the loop is

$$\mathbf{m} = \hat{\mathbf{n}}NIA = \hat{\mathbf{n}}20 \times 10 \times (30 \times 10) \times 10^{-4} = \hat{\mathbf{n}}6 \quad (\text{A·m}^2),$$

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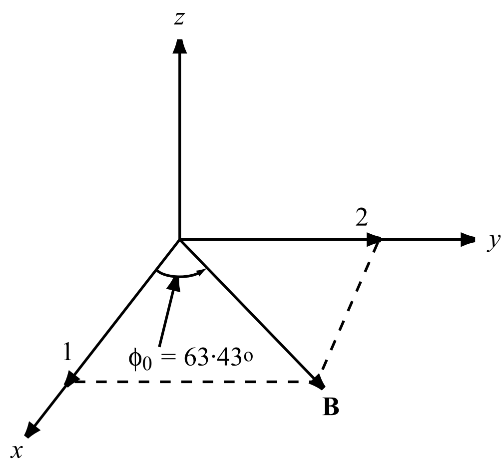

Fig 4. (a) Direction of **B**.

where $\hat{\mathbf{n}}$ is the surface normal in accordance with the right-hand rule. When the loop is in the negative-*y* of the *y*–*z* plane, $\hat{\mathbf{n}}$ is equal to $\hat{\mathbf{x}}$, but when the plane of the loop is moved to an angle ϕ, $\hat{\mathbf{n}}$ becomes

$$\begin{aligned}
\hat{\mathbf{n}} &= \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi, \\
\mathbf{T} = \mathbf{m}\times\mathbf{B} &= \hat{\mathbf{n}}\,6 \times 2\times 10^{-2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}2) \\
&= (\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi)\,6 \times 2\times 10^{-2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}2) \\
&= \hat{\mathbf{z}}0.12[2\cos\phi - \sin\phi] \quad (\text{N}\cdot\text{m}).
\end{aligned}$$

(b) The torque is zero when

$$2\cos\phi - \sin\phi = 0,$$

or

$$\tan\phi = 2, \qquad \phi = 63.43^\circ \text{ or } -116.57^\circ.$$

Thus, when $\hat{\mathbf{n}}$ is parallel to $\mathbf{B}$, $\mathbf{T} = 0$.

(c) The torque is a maximum when $\hat{\mathbf{n}}$ is perpendicular to $\mathbf{B}$, which occurs at

$$\phi = 63.43 \pm 90^\circ = -26.57^\circ \text{ or } +153.43^\circ.$$

Mathematically, we can obtain the same result by taking the derivative of **T** and equating it to zero to f nd the values of ϕ at which $|\mathbf{T}|$ is a maximum. Thus,

$$\frac{\partial T}{\partial\phi} = \frac{\partial}{\partial\phi}(0.12(2\cos\phi - \sin\phi)) = 0$$

or

$$-2\sin\phi + \cos\phi = 0,$$

which gives $\tan\phi = -\frac{1}{2}$, or

$$\phi = -26.57^\circ \text{ or } 153.43^\circ,$$

at which $\mathbf{T} = \hat{\mathbf{z}}0.27$ (N·m).