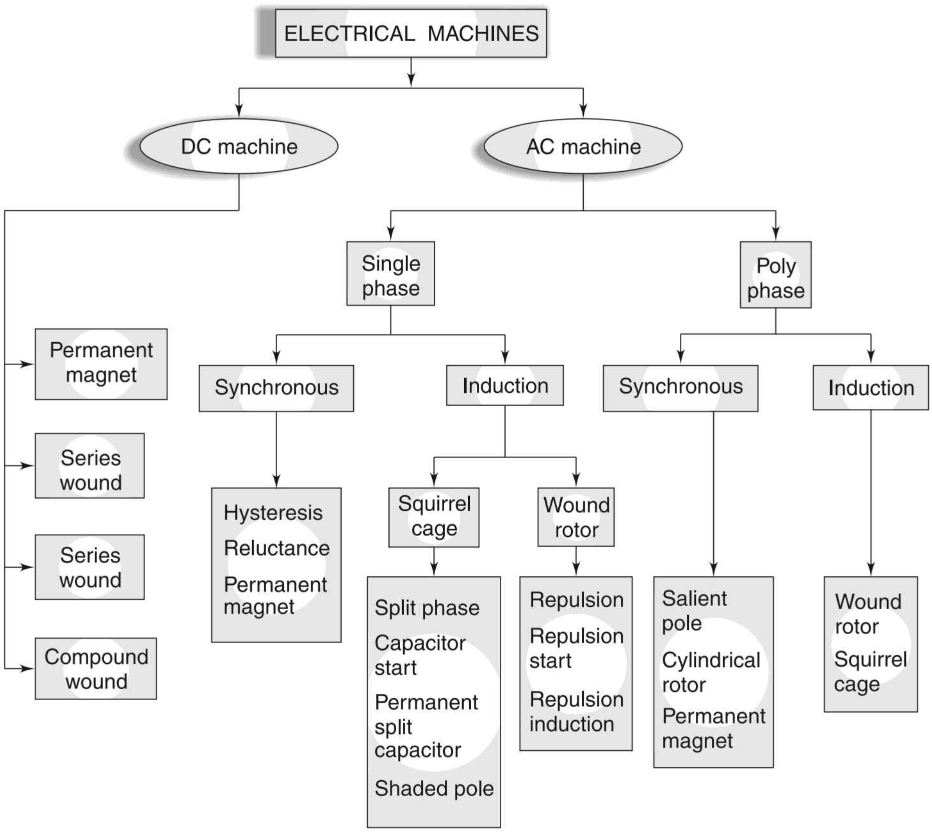
1 INTRODUCTION

Electrical machine is simply an energy converting device in which two electrical circuits are coupled by means of a magnetic circuit. Most people, not only in industrialized countries, are heavily dependent on electrical machines in their daily lives. Electrical machines can be classified according to their construction, type, use and performance. Their general classification is as under.



In this chapter, we will discuss synchronous machines, including their construction, working principle, performance parameters, application etc.

Any electrical machine which rotates at a speed fixed by the supply frequency and the number of poles is called synchronous machine. Like other rotating machines, synchronous machine is an important electromechanical energy converter. Synchronous generators (also called alternators) are connected in parallel to form a large power system to supply energy to a variety of loads. For such purposes, very large synchronous machines are built with ratings of 10 MW to 100 MW.

Synchronous machines can also be used as motor. Synchronous motor operates at a precise speed called its synchronous speed, and hence it is called a constant speed motor. If we compare it with induction motor, we find that the induction motor operates at lagging power factor, while the synchronous motor has a variable power factor characteristic and hence it is used for power factor correction application also.

If the synchronous motor operates without any mechanical load, it is called compensator. It works like a variable capacitor when its field is overexcited and works like a variable inductor when its field is under-excited. It is often used in power system to control reactive power.

Like DC machines, synchronous machine also have one electric circuit or armature winding and other magnetic circuit where flux is produced. In DC machines, the armature rotates and the field circuit remains stationary, but in a synchronous machine, the armature (stator) remains stationary and the field circuit rotates. The stator consists of a number of slots in its inner periphery wherein armature windings are placed. The rotor is like a fly wheel having alternate *N* and *S* poles on its outer rim. These poles are magnetized by a DC source. Generally, this DC source is obtained by DC shunt generator which is mounted on the shaft of the alternator itself. This DC power is supplied through slip rings and brush gears. Recently, brushless excitation system has been developed in which 3-phase AC exciter and rectifiers feed DC to the alternator. When the rotor rotates, the armature conductors are cut off by the flux inducing emf in them. Since the magnetic poles are changing on the rotor, an alternating emf is produced in the stator conductors.

A synchronous machine may be single-phase, two-phase or three-phase type. For a single-phase machine, all the armature coils are connected so that the individual voltage of each coil is taken out from a pair of output terminals. For two phases, a set of armature windings is placed so that outputs from three terminals having one common terminal are 90° out of phase. For three phases, a set of three windings are placed so that output from three terminals with a neutral phase difference of 120° are available.

Synchronous machines may be classified on the basis of their construction as (i) rotating armature type and (ii) rotating field type. However, for the generation of emf, it is immaterial whether the conductor moves across the field or vice versa.

The rotating armature type alternator is similar to the DC generator except that there are four slip rings in place of the commutator to collect the current. Field is produced by stationary field circuit. This is suitable for small low-voltage generators (up to about 200 or 250 kVA) because of small current. All the medium and large size alternators are always constructed with revolving field.

.2 ADVANTAGES OF STATIONARY ARMATURE

(i) Output current can be drawn directly from the fixed terminal of the stator and fed to the load without passing through the gear and brushes. Therefore, load circuit can be directly connected to the fixed terminals.
(ii) Stationary armature winding can be easily placed in a rigid frame.
(iii) Armature winding can be cooled more easily because air space or cooling ducts can be large.
(iv) If stator winding is stationary, it is very easy to insulate for high AC voltage as 33 kVA or more.
(v) Only two slip rings are required to supply DC to the field circuit.
(vi) The brush and the gears are on low power side (field circuit), so are easy to insulate because the construction is light.
(vii) As the construction of the rotor is simple, higher speed can be obtained which provide increased output.
(viii) Maintenance of stationary stator winding is easy.

.2.1 Comparison between Rotating and Stationary Field Systems

Sl No.	Rotating pole	Stationary pole
01	Field system is rotating.	Field system is stationary.
02	Armature is stationary.	Armature is rotating.
03	Two slip rings are required.	Four slip rings are required.
04	Slip ring current rating is 5% of the total current.	Slip ring current rating is 95% of the total current.
05	Sparking is weak.	Sparking is very strong.
06	Brush friction is low.	Brush friction is high.
07	Brush drop is not substantial.	Brush drop is substantial.
08	It is used for above 25 kVA.	It is used for below 25 kVA.
09	It is used for voltage above 6.6 kV to 30 kV	It is used for below 6.6 kV

3 SPEED AND FREQUENCY

According to the definition of synchronous machines, the frequency of the generated emf depends upon the number of field poles and the speed at which the field poles rotate. A complete cycle of voltage is generated when coil passes over the pair of field poles.

Let P be the total number of field poles

p the pair of field poles

N the speed of the field poles in rpm

n the speed of the field poles in rps.

f the frequency of the generated voltage in Hz.

Then, obviously, $\frac{N}{60} = n$ and $\frac{P}{2} = p$

In one revolution of the rotor, armature coil is cut by $P/2$ north poles and $P/2$ south poles. So, when an armature coil passes over a pair of field poles, one cycle is generated. So, the number of cycles generated in one revolution of the rotor will be equal to the number of pole pairs.

So the number of cycles in one revolution = p

And the number of revolutions per second = n

Now, Frequency = Number of cycles per second

So, Frequency = $\dfrac{\text{Number of cycles}}{\text{Revolutions}} \times \dfrac{\text{Revolutions}}{\text{Seconds}} = p \times n$

Thus,
$$f = \frac{PN}{120} \qquad (3.1)$$

From this equation, it is clear that rotor speed has a constant relationship with field poles and the frequency of the generated voltage in the armature coil. From this equation, synchronous speed can be given by

$$\text{Synchronous speed, } N_s = \frac{120f}{P} \qquad (3.2)$$

A machine which runs at a synchronous speed is called synchronous machine.

Example .1 A 20-pole alternator runs at 300 rpm. Find the frequency of the voltage generated by the generator. If the speed is 360 rpm, what will be the frequency of the voltage?

Solution: Synchronous speed is given as $N_s = 120\dfrac{f}{P}$

$$\therefore \quad f = \frac{PN_s}{120} = \frac{20 \times 300}{120} = 50 \text{ Hz}$$

If the speed is 360 rpm, $f = \dfrac{20 \times 360}{120} = 60$ Hz

Example .2 A synchronous motor generator set is running to link up a 30 Hz with a 50 Hz system. If the motor has 8 poles, find the speed of the set and the number of poles on the alternator.

Solution: The synchronous motor has poles and frequency of supply is 30 Hz.

$$\therefore \quad N_s = 120\frac{f}{P} = 120 \times \frac{30}{8} = 450 \text{ rpm.}$$

The alternator is coupled to the motor so it runs on the same speed, i.e., 450 rpm and it has to supply at 60 Hz.

$$\therefore \quad P = 120 \times \frac{60}{450} = 16 \text{ poles}$$

.4 CONSTRUCTION OF SYNCHRONOUS MACHINES

Construction of the synchronous generator (also called alternator) and synchronous motor is same. Same machine can be used as an alternator or a motor.

Synchronous machine has the following two main parts.

(i) Stator or armature.

(ii) Rotor or field circuit.

4.1 Stator (Armature)

Stationary part of the synchronous machine is called stator or armature. This consists of a core and a number of slots on the inner periphery to hold the armature winding just like the armature of a DC generator. A number of laminations made from special steel stampings and insulated from each other by using a varnish or paper, are used in stator core. Such laminations are used to reduce eddy current losses because the field winding is rotating; the flux is cut by the armature core also. Generally, steel is used to reduce hysteresis losses. The sectional view of armature is shown in Fig. .1.

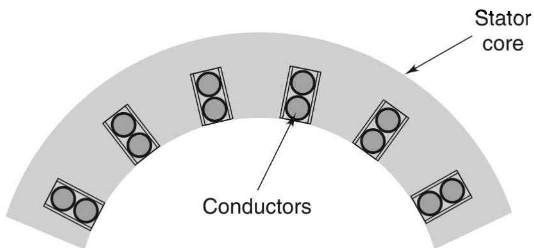


Fig. 1 Section of alternator

The entire core is fabricated from steel plates. As said above, the core has slots on its inner periphery to hold armature conductors. Different types of slots are shown in Fig. .2.

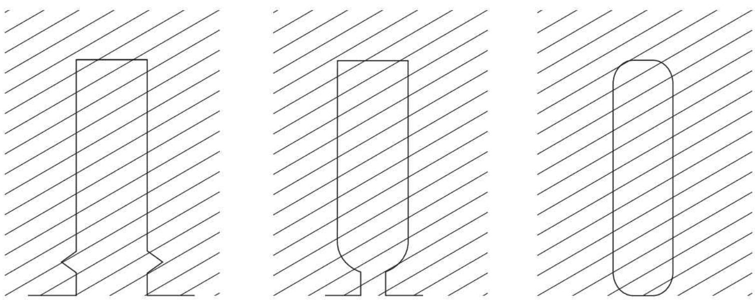


Fig. .2 Different types of slots

Open type slots are widely used because form wound coil can be easily placed in them and also can be removed at the time of repairing. It has a disadvantage that it distributes air gap flux into bunches that produces ripples in the generated emf. The semi-closed slots are good in this respect but they do not allow the form wound coil. Closed slots do not disturb the air gap flux but have disadvantages, like: (i) they increase the inductance of the windings, (ii) conductors have to be threaded resulting in more labor cost, and (iii) they create problems in end connections. Hence, they are rarely used.

During the machine operation, the frame itself does not carry any flux but gives support to the core. Holes are provided in the frame for ventilation purpose.

4.2 Rotor

According to their construction, there are two types of rotors which are used in the synchronous alternator.

(a) Salient-pole type rotor

This is also known as projected-pole type because looking from the surface of the rotor all the poles appear projecting (jutting) out. The poles are made of thick steel laminations. They are generally bolted to the rotor as shown in Fig. .3. A specific shape is made at the pole face so that radial air gap length is increased from the pole centre to the pole tips to produce sinusoidal emf. The field winding is placed on the pole shoe. Generally, these rotors are available with large diameter and short axial length, because the centrifugal force which is acting on the rotating member of the machine limits the size of the rotor. Due to low mechanical strength, the salient-pole type is preferred for slow speed alternators whose range is from 125 rpm to 500 rpm. Generally, water turbines and IC engines are used as prime movers for such rotors.

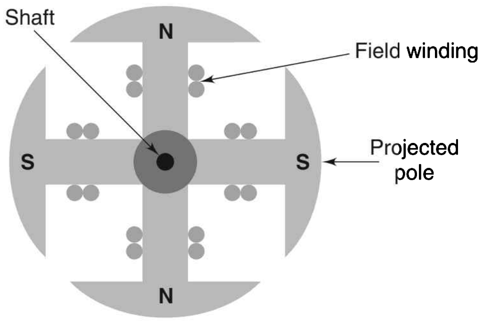


Fig. .3 Cross section view of salient pole rotor

(b) Cylindrical rotor

This type of rotor is also known as non-salient type or non-projected pole type, or round rotor. Figure 4 shows smooth cylindrical type of rotor.

The rotor has a small solid steel cylinder. This cylinder has a number of slots on its outer periphery for holding the field coil. Steel or manganese wedges are used to cover the top of the slot. The portion which is not slotted also acts as a pole. Here, the poles do not project out and the rotor surface is smooth which provides uniform air gap between stator and the rotor.

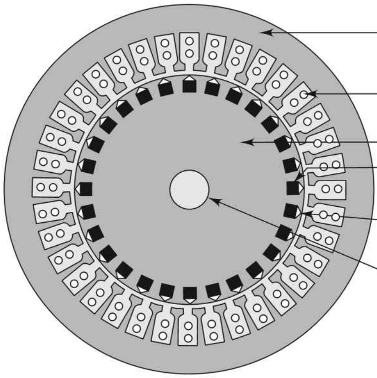


Fig. 4 Cross-sectional view of cylindrical rotor machine

These rotors are generally available with small diameters and long axial lengths to limits the peripheral speed. The main advantage of this rotor is that it has strong mechanical strength and hence is used for high speed alternators with range from 1500 to 3000 rpm. Such high speed alternators are called turbo alternators. Generally, steam turbines and electric motors are used as a prime mover for such rotors.

4.3 Difference between Salient-Pole Alternator and Cylindrical Alternator

Sl. No.	*Salient-pole alternator*	*Cylindrical alternator*
01	It uses damper winding.	Does not use damper winding.
02	Rotor surface is not smooth.	Rotor surface is smooth.
03	Air friction is large.	Air friction is minimal.
04	Speed range is 150 to 1500 rpm	Speed is 1500 or 3000 rpm
05	Efficiency is lower than that of non-salient pole.	Efficiency is higher than that of salient pole.
06	Armature reaction is complex.	Armature reaction is simple.
07	It is used in hydro, diesel power station.	It is used in thermal and nuclear power station.
08	Voltage regulation is better than in non-salient.	Voltage regulation is good.
09	It has large diameter and short axial length.	It has short diameter and long axial length.
10	The poles are projected out from the rotor surface.	The poles are not projected out but have smooth rotor surface.
11	It has a non-uniform air gap.	It has a uniform air gap.

5 AC WINDING DESIGN

Separate coils may be connected in certain methods. The two most common methods used for are Lap and Wave winding. The winding used in rotating electrical machine can be classified as under:

.5.1 Concentrated Winding

Such winding is provided where the number of slots is equal to the number of poles. In this winding, all the turns have the same magnetic axis. Simple form of this winding is shown in Fig. `.5. It is called Skelton wave winding.

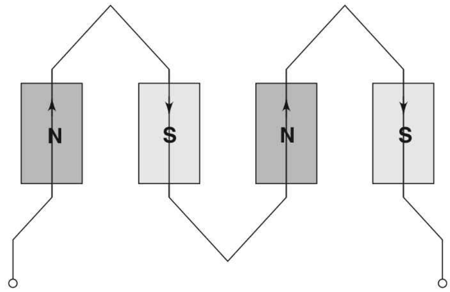


Fig .5 Skelton wave winding

In this winding, induced emf is maximum but the wave form is not exactly sinusoidal. This winding requires much space for end connections of the coil so this is rarely used in practice.

If single-turn coil windings are replaced by multi-turn coils to get higher value of emf, the multi-turn half coil winding is obtained as in Fig. .6. Here, coils only cover the one half of the periphery of the armature. They are also called half-coiled or hemitropic winding (Fig. 6).

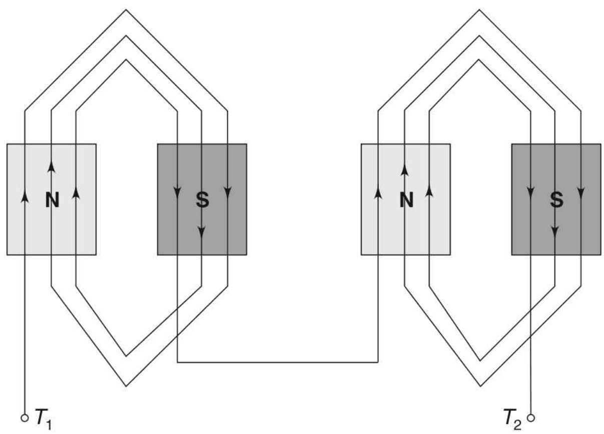


Fig. .6 Half coiled or hemitropic winding

If the coil is distributed over the whole armature, it is called whole-coiled winding.

Examples of concentrated windings are:

(i) Field winding for salient pole synchronous machine.
(ii) DC machine winding.
(iii) Primary and secondary winding of transformer.

Concentrated winding is not generally used because of the following drawbacks:

Drawbacks

- Pure sine wave is not produced.
- More conductors required.
- Winding becomes crowding resulting in poor heat dissipation.

5.2 Distributed Winding

If conductors are placed in different slots under one pole, the winding is called distributed winding. This winding may be partially or completely distributed. It reduces the emf but is used most commonly because of the following advantages:

- Harmonic emfs are reduced.
- It reduces the effect of armature reaction and armature reactance.
- High density copper can be used for even distribution of copper loss therefore gives efficient cooling.
- The core is better used.

This distributed winding also may be lap, wave and spiral winding. Lap windings are used in high speed synchronous machine stators. Wave winding is used for rotors of wound rotor type induction motors, whereas spiral winding is used in slow speed, large diameter machines.

Examples of the distributed windings are:

(i) Stator and rotor of induction motor.
(ii) Armatures of both synchronous and DC machines.

6 AC ARMATURE WINDING

In general, various types of armature windings are used as per requirement and applications. These are described as under:

6.1 Closed Winding

This is a closed path winding, i.e., wherever you may start, the winding traverses the path and takes you back to the starting point.

Examples: It is used only for DC machines and AC commutator machines.

6.2 Open Winding

This winding terminates at suitable number of slip rings or terminals.

Such winding is used only for AC machines, like synchronous and induction machines.

It is very essential to know some terms before studying the winding.

Conductor

A length of wire that takes part in energy conversion is called conductor.

Turn

One turn consists of two conductors which are connected in series so that the emf induced in them is double.

Coil

When one or more turns are connected in series and the two ends of them are connected to adjacent commutator segment, it is called a coil. A coil may have any number of turns.

Coil side

Each coil has two sides which are placed in two different slots near a pole pitch.

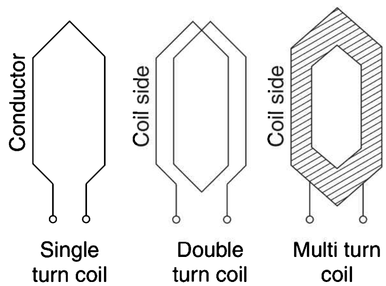


Fig. 7

Coil group

A coil group may have one or more single coils. It is the product of the number of phases and the number of poles.

Pole pitch

It is defined as the number of conductors per pole. If there are 54 conductors for 6 poles then the pole pitch will be 54/6, i.e., 9 conductors per pole. Or, it is also given by the distance between the centres of two adjacent poles. The pole pitch is always equal to 180° electrical.

Relation between electrcial and mechanical degrees

Mechanical degree is the angle between two points based in their mechanical or physical position. Electrical degree is the angle between two points in the rotating electrical machine. The entire electrical machine operates with the help of a magnetic field. So, electrical degree is in reference to the magnetic field. The distance between the adjacent North and South poles, i.e., pole pitch is 180 electrical degrees or angle.

If P is the number of poles or p is the number of pole pairs, electrical degree θ_e with reference to the mechanical degree θ_m is given by

$$\theta_e = \frac{P}{2}\theta_m \text{ or } p\,\theta_m \tag{6.1}$$

Coil pitch

The distance between the two sides of the coils is called coil pitch or coil span. It is usually measured in terms of slots, teeth or electrical degrees.

6.3 Single-layer Winding

In this winding, one coil side occupies the total slot area as shown in Fig. 8. This winding is used in only small AC machines. It may be concentric, lap or wave winding.

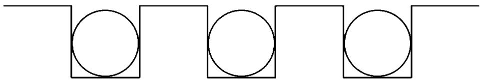


Fig. 8 Single-layer winding

Advantages of single-layer winding

(i) It gives higher efficiency and quieter operation because of narrow slot opening.
(ii) Space factor for slot is higher because of absence of inter-layer separator.

6.4 Double-layer Winding

In this winding, slots contain even number (maybe 2, 4, 6, ...) coil sides in two layers as shown in Fig. 9.9. This winding is more commonly used above 5 kW machines. Synchronous machine armature and induction motor stators above a few kW are wound with double-layer winding.

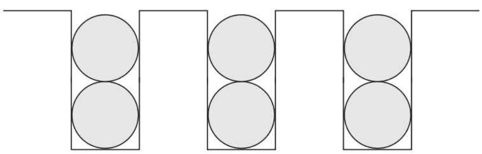


Fig. 9 Double-layer winding

6.5 Advantages of Double-layer Winding Over Single-layer

- Easier to manufacture and cost of the coil is low.
- Fractional slot winding can be used.
- Short pitched coil winding can be done.
- Lower leakage reactance increases performance of the machine.
- Better emf waveform in case of generator.

6.6 Chorded Coil

If the coil span and pole pitch are equal, or say equal to the180° electrical, the winding is called full-pitched winding. In this winding, one side of the coil is under N pole and the other is under S pole. The induced emf differs by 180° in phase but the coil is connected in such a way that emfs add up to give the resulting emf *E* as in Fig. 10.

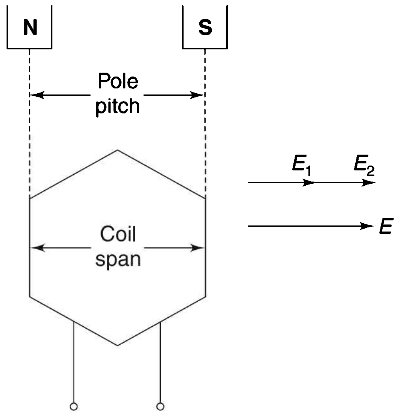


Fig. 10 Full pitched coil

If the coil span is less than the pole pitch, or say less than180° electrical, the winding is called short-pitched winding. In this winding, induced emf in each coil is not in phase. Hence, the resultant emf is the phasor sum of induced emf in the coil side which is slightly less than their arithmetic sum. This is shown in Fig. 11.

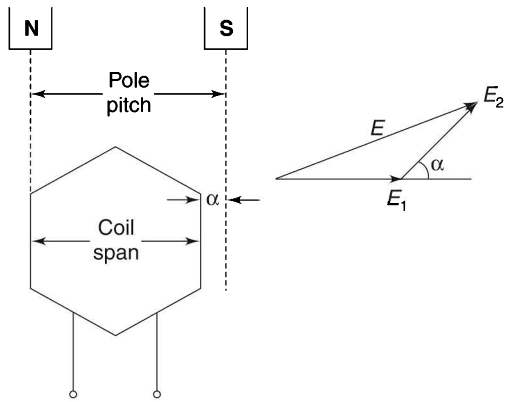


Fig. 11 Short pitched coil

Hence, induced emf in this winding is slightly less than full-pitch winding under the same conditions.

6.7 Advantages of Short Pitched Winding

- Waveform of the induced emf is improved and the distorting harmonic can be reduced or totally removed.
- Saving of copper due to shorter span. Also, reduced copper loss.
- Reduction in inductance of the winding due to lesser length of the coil ends.
- Fractional number of slots per pole can be used which reduces tooth ripples.
- Mechanical strength of the coil is increased.

6.8 Integral Slot Winding

To get better performance of the alternator, flux should be uniform throughout the surface of the armature. For this purpose, the armature conductors should be properly distributed. The number of slots in an AC machine should always be an integral multiple of three. When the number of the slots per pole per phase (m) is an integer, the winding is called integer slot winding. For example, a 3-phase winding with 36 slots and 4-poles is integral slot winding because $m = \dfrac{36}{3 \times 4} = 3$, is an integer.

6.9 Fractional Slot Winding

When the number of slots is fractional, the winding is called fractional slot winding. In both cases, the number of slots per phase must be a whole number. For example, a 3-phase winding with 30 slots and 4-poles is a fractional slot winding because $m = \dfrac{30}{3 \times 4} = \dfrac{5}{2}$ is not an integer. Fractional slot winding is practicable only with double layer arrangement.

Advantages of fractional slot winding are: (i) it reduces the parallel circuit, (ii) it is easy to manufacture, (iii) it is flexible, (iv) it has low mmf harmonics, leakage reactance and cost of copper.

The main drawback of the fractional slot winding is that it is a little complicated.

6.10 Integral-Slot Chorded Winding

When the coil span is less than pole pitch, the winding is called integral-slot chorded winding. In this winding, the coil span generally changes from 2/3 pole pitch to full pitch.

Advantages

The advantages of using chorded coil are:

- The copper required for the end connection is less.
- It reduces the harmonic of phase emf and mmf.

7 FACTORS GOVERNING ARMATURE WINDING

(i) The pole pitch must be equal to coil span.
(ii) The coils must be connected in such a way that the emfs induced in them help each other.
(iii) The winding may be single layer or double layer.
(iv) Winding must be designed in such a way that it produces sinusoidal emf.

In poly-phase winding it is necessary that:

(i) the generated emf in the all the phases are must be equal magnitude.
(ii) the phase emfs have identical waveform.
(iii) the phase emfs have the same frequency.

Some important terms related to poly-phase windings are:

Balanced winding

In balanced winding the number of coils per coil group is a whole number. Each pole must have equal number of coils of different phases.

Unbalanced winding

In unbalanced winding the number of coils per coil group is not a whole number. Poles have unequal number of coils of different phases.

Phase belt or phase band

It is defined as the group of adjacent slots belonging to one phase under one pole pair.

> It is important to note that the winding is either balanced or unbalanced, each phase must have equal number of coils. Also, the number of coils is always equal to the number of slots.

Poly-phase windings are like single-phase winding; the only difference is that in the two-phase alternator, there are two separate single-phase windings placed 90° apart and in the 3-phase type alternator, there are three single-phase windings placed 60° electrically apart for convenience.

Like DC armature winding, poly-phase armature windings are usually double layer winding. Further, poly-phase armature windings can be lap winding or wave winding just like the DC armature winding. The armature of the synchronous machine is generally lap wound and the rotor of the medium and large size induction motor is wave wound. The phase spread of *m*-phase windings are usually 180°/*m*; for 3-phase it is 60°.

.8 WINDING FACTOR

The product of pitch factor k_c (**also called** coil span factor) and Distribution factor k_d is known as winding factor.

$$k_w = k_c\, k_d \qquad (`8.1)$$

.9 PITCH FACTOR

In a short-pitch winding, the induced emfs in the two sides of the coil are not in a phase. So, their resultant emf, given by the phasor sum, is always less than its arithmetic sum. The ratio of phasor-sum generated emfs per coil to the arithmetic sum of generated emfs per coil is known as pitch factor.

Pitch factor, $k_c = \dfrac{\text{Phasor sum of coil side emfs}}{\text{Arithmatic sum of coil side emfs}}$

Pitch factor is always less than unity.

Let the coil having a pitch short by an angle α electrical space degree form full pitch and induced emf in each coil side be E as shown in Fig. . .12. Now, if the coil is full pitched, the total induced emf on each coil side will be $2E$. If the coil is short pitched by an angle α, the resultant emf, E_R will be the phasor sum of two voltages α^0 apart.

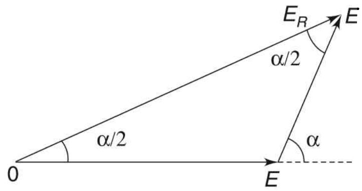


Fig. . .12 Phasor diagram for pitch factor

$$E_R = 2E \cos\frac{\alpha}{2}$$

$$\text{Pitch factor} = \frac{2E\cos\frac{\alpha}{2}}{2E} = \cos\frac{\alpha}{2}$$

$$\text{Pitch factor, } k_c = \cos\frac{\alpha}{2} \qquad (\text{: } 9.1)$$

If the coil span is reduced by one slot, then the phase angle, α, can be given as $180°/n$. where n is the number of the slot per pole. If proper short pitching angle is selected, the harmonics can be removed. If N is the harmonic order, i.e., 3rd, 5th, 7th, harmonic etc, the pitch factor will be $k_{cN} = \cos\frac{N\alpha}{2}$. k_c will be zero when the term $\frac{N\alpha}{2} = 90°$.

10 DISTRIBUTION FACTOR

If all the coil sides of any phase under one pole are bunched in one slot, winding is called concentrated winding. Then the total induced emf is the arithmetic sum of all the coils under that pole.

However, to get a better wave shape, the coils are not bunched in one slot but are distributed over a number of slots. Such a winding is called distributed winding. In this winding, coils per phase are displaced from each other by a certain angle equal to the slot pitch. (The slot pitch is defined as the centre of the one slot to the centre of the adjacent slot.) So the total voltage induced in any phase is the phasor sum of the individual coil voltages.

Distribution factor is defined as the ratio of the phasor sum of the emf produced in the coils under one pole to the arithmetic sum of the emf produced in all the coils. It is also known as breath factor (K_b) or distribution factor (K_d).

Let the number of slots per pole be n

Number of slots per pole per phase (i.e., slots per phase belt) = $m = \dfrac{\text{Slots}}{\text{Poles} \times \text{Phase}}$

Induced emf in each coil side = E

Angular displacement between the adjacent slots in electrical degree = $\beta = \dfrac{180°}{\text{Slots/Pole}}$ $= \dfrac{180° \times \text{Poles}}{\text{Slots}}$

Thus, one phase of the winding having coils is arranged in m consecutive slots. Voltages E_1, E_2, E_3, E_4 are the individual coil voltages. Each coil voltage is out of phase with the next coil voltage by the slot pitch β. Voltage polygon for induced emf is shown in Fig. 9.13 for four coils of a group (m = 4). Voltages E_1, E_2, E_3, E_4 are represented by AB, BC, *CD* and DF. Each of this phasor is a chord of a circle with centre O. The phasor sum AF gives the resultant voltage E.

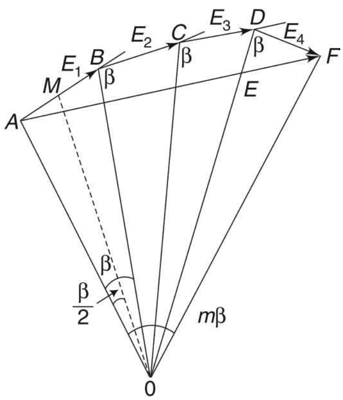


Fig. .13 Distribution factor

Arithmetic sum of coil voltages = $m 2\ \text{OA} \sin \dfrac{\beta}{2}$

The resultant emf induced in phase will be the phasor sum given by the phasor AF, as in Fig. ` 13.

Resultant emf, $$E = \text{AF} = 2\ \text{OA} \sin \frac{m\beta}{2}$$

So, distribution factor, $$K_d = \frac{\text{Phasor sum of emf}}{\text{Arithmetic sum of emf}} = \frac{2\ \text{OA} \sin \frac{m\beta}{2}}{m 2\ \text{OA} \sin \frac{m\beta}{2}} = \frac{\sin \frac{m\beta}{2}}{m \sin \frac{m\beta}{2}}$$

$$\text{Distribution factor, } K_d = \frac{\sin \frac{m\beta}{2}}{m \sin \frac{m\beta}{2}} \qquad (\ 10.1)$$

It may be noted that distribution factor depends only on the number of slots under a given pole. It does not depend on the type of winding, lap or wave, or the number of turns per coil, etc. As the number of slots per pole is increased, the value of distribution factor decreases. The value of distribution factor is also less than unity; being about 0.9. Further, when the term $\sin \frac{mN\beta}{2} = 180°$, the k_d will be zero, where N is the harmonic order.

11 EQUATION OF INDUCED EMF

Consider an alternator in which all the coils used for armature winding are full pitched and all the conductors are placed in a single slot, i.e., it is a concentrated winding.

Let Z be the number of conductors or coil sides per phase

$= 2T$ (where T is the total number of conductors or turns per phase and each turn has 2 sides.)

P be the number of poles.

f = Frequency of the induced emf, in Hz.

ϕ = Flux per pole, in Wb.

K_d = Distribution factor, and K_p = Pitch factor.

K_f = Form factor = 1.11, if emf is assumed sinusoidal.

N = Rotor rpm.

In one revolution, each stator conductor is cut by a flux of ϕP in webers.

So, $d\phi = \phi P$, and $dt = 60/N$ second.

So, average emf induced per conductor $= \frac{d\phi}{dt} = \frac{\phi P}{60/N} = \frac{\phi NP}{60}$

Average emf induced per conductor $= \frac{\phi P}{60} \times \frac{120f}{P} = 2f\phi$ volt

$\left(\text{Putting the value of } N = \frac{120f}{P}\right)$

If there are Z conductors in coil per phase,

Average emf per phase $= 2f\phi Z = 4f\phi T$ volt (because $Z = 2T$)

RMS value of emf per phase $= 4.44 f\phi T$ volt.

$$\text{RMS value of emf per phase} = 4.44 f\phi T \qquad (11.1)$$

This equation is same as the equation of induced emf in the transformer.

Considering the coil span factor and distribution factor, the actual per phase voltage generated is given by

$$E_P = 4.44\, k_c k_d f \phi T \qquad (11.2)$$

Equation (11.2) is known as the complete emf equation of the alternator. Sometimes, ($k_c k_d\, T$) is known as effective turns per phase. Its value is always smaller than the actual number of turns per phase because of fractional pitch coils and distribution of winding over slots under each pole.

Example .3 A 3-phase, 12-pole, 500 rpm star-connected alternator has 144 slots with 10 conductors per slot. The flux per pole is 0.055 Wb. Find the line voltage and phase voltage.

Solution: Considering the full pitched winding:

$$\alpha = 0°, \text{ hence } k_c = \cos\frac{\alpha}{2} = \cos 0° = 1$$

The slots per phase per pole (phase band), $m = \dfrac{\text{Slots}}{\text{Poles} \times \text{Phase}} = \dfrac{144}{12 \times 3} = 4$

$$\beta = \frac{180° \times \text{Poles}}{\text{Slots}} = \frac{180° \times 12}{144} = 15$$

Distribution factor, $k_d = \dfrac{\sin m\,\beta/2}{m \sin \beta/2} = \dfrac{\sin 4 \times 15/2}{4 \sin 15/2} = \dfrac{\sin 30°}{4 \sin 7.5°} = 0.9578$

Total number of conductors = Conductors per slot × Number of slots = 10 × 144 = 1440

Conductor per phase, $Z = 2T = \dfrac{1440}{3} = 480$, hence $T = 240$

Voltage generated per phase, $E_p = 4.44\, k_c k_d f\, \phi T$

$$\therefore \quad E_p = 4.44 \times 1 \times 0.9578 \times 50 \times 0.055 \times 240 = 2806 \text{ V}$$

$$E_L = \sqrt{3} E_P = \sqrt{3} \times 2806 = 4860 \text{ V}$$

Example .4 A 3-phase, 8-pole alternator has star-connected stator and its speed is 900 rpm. The stator has 120 slots and 8 conductors per slot. The flux per pole is 0.094 Wb and it is distributed sinusoidally. Find the voltage generated by the machine if the winding factor is 0.97.

Solution:

$$f = \frac{PN_s}{120} = \frac{8 \times 900}{120} = 60 \text{ Hz}$$

Total number of conductors per phase, $T = \dfrac{120}{2} \times \dfrac{8}{3} = 160$

Generated voltage per phase, $E_p = 4.44\, k_w f\, \phi T$

$$E_P = 4.44 \times 0.97 \times 60 \times 0.094 \times 160 = 3885 \text{ V}$$

Line voltage, $E_L = \sqrt{3} E_P = 6730 \text{ V}$

Example .5 A 3-phase, 18-pole, alternator has a resultant flux of 0.093 Wb per pole. The flux is uniformly distributed over the pole. The stator has 4 slots per phase per pole and 8 conductors per slot are connected in double layers. The coil span is 150° electrical. Calculate the phase and line voltage if the speed is 400 rpm.

Solution:

$$f = \frac{PN_s}{120} = \frac{18 \times 400}{120} = 60 \text{ Hz}$$

$$\alpha = 180° - 150° = 30°$$

$$k_c = \cos\frac{\alpha}{2} = \cos 15° = 0.9659$$

$$m = \text{slot per pole per phase} = \frac{\text{Slots}}{\text{Poles} \times \text{Phase}}$$

Slots = m × Poles × Phase = 4 × 18 × 3 = 216

Total conductors per phase, $T = \dfrac{216 \times 8}{2 \times 3} = 288$

$$\beta = \frac{180° \times \text{Poles}}{\text{Slots}} = \frac{180 \times 18}{216} = 15°$$

$$k_d = \frac{\sin m\,\beta/2}{m \sin \beta/2} = \frac{\sin 4 \times 15/2}{4 \sin 15/2} = \frac{\sin 30°}{4 \sin 7.5°} = 0.9578$$

$$E_P = 4.44\, k_c k_d f\,\phi T = 4.44 \times 0.9659 \times 0.9578 \times 60 \times 0.093 \times 288 = 6601 \text{ V}$$

$$E_L = \sqrt{3}E_P = 11433 \text{ V}$$

Example .6 A 3-phase, 50 Hz, 4-pole, star-connected alternator has 72 slots with 4 conductors per slot. The coil span is 2 coils less than pole pitch. If the machine gives 6600 V between lines on an open circuit, determine the useful flux per pole.

Solution:

$$m = \frac{\text{Slots}}{\text{Poles} \times \text{Phase}} = \frac{72}{4 \times 3} = 6$$

No. of slots per pole = $\dfrac{72}{4} = 18$, hence, slot angle, $\beta = \dfrac{180}{18} = 10°$

Coil pitch = 16 Slots angles = 16 × 10 = 160°

$$\alpha = 180° - 160° = 20°$$

$$k_c = \cos\frac{\alpha}{2} = \cos 10° = 0.9848$$

$$k_d = \frac{\sin m\,\beta/2}{m \sin \beta/2} = \frac{\sin 6 \times 10/2}{6 \sin 10/2} = \frac{\sin 30°}{6 \sin 5°} = 0.9562$$

Total number of conductors per slot per phase = $\dfrac{72 \times 4}{3 \times 2} = 48$

Now, $E_P = 4.44\, k_c k_d f\,\phi T$

$$\frac{6600}{\sqrt{3}} = 4.44 \times 0.9848 \times 0.9562 \times 50 \times \phi \times 48$$

$$\therefore \quad \phi = \frac{6600}{\sqrt{3} \times 4.44 \times 0.9848 \times 0.9562 \times 50 \times 48} = 0.3797 \text{ Wb}$$

Example .7 A 2-pole, 3-phase, 50 Hz, star-connected alternator has 72 slots and 4 conductors per slot. The armature winding is of double layer winding. Coils are short pitched in such a way that if one coil side is placed in slot number 1 then the other is placed in slot number 13. Find the useful flux per pole required to generate line voltage 3300 V.

Solution: No. of slots per pole $= \frac{72}{2} = 36$

Slot angle, $\beta = \frac{180°}{\text{slot per pole}} = \frac{180}{36} = 5°$

Coil span = $12 \times \beta = 12 \times 5 = 60$

$$\alpha = 180° - 60° = 120°$$

$$k_c = \cos\frac{\alpha}{2} = \cos 60° = 0.5$$

$$m = \frac{72}{2 \times 3} = 12$$

$$k_d = \frac{\sin m\beta/2}{m \sin \beta/2} = \frac{\sin 12 \times 5/2}{12 \sin 5/2} = \frac{\sin 30°}{12 \sin 2.5°} = 0.9552$$

Total number of conductors per slot per phase $= \frac{72 \times 4}{3 \times 2} = 48$

Now, $E_P = 4.44\, k_c k_d f\, \phi T$

$$\frac{3300}{\sqrt{3}} = 4.44 \times 0.5 \times 0.9552 \times 50 \times \phi \times 48$$

$\therefore$ $$\phi = \frac{3300}{\sqrt{3} \times 4.44 \times 0.5 \times 0.9552 \times 50 \times 48} = 0.3743 \text{ Wb}$$

.12 MMF OF CONCENTRATED WINDING

Now, let us see what happens when the field winding of the alternator is excited, and what type of mmf and flux are produced in the air gap of the alternator. The reason is that these factors greatly affect the emf induced in the alternator. Here, it should be noted that the flux density in the air gap of the synchronous machine is neither uniform nor non-sinusoidal due to various reasons. Here, we first see the mmf produced in the concentrated winding and then distributed winding.

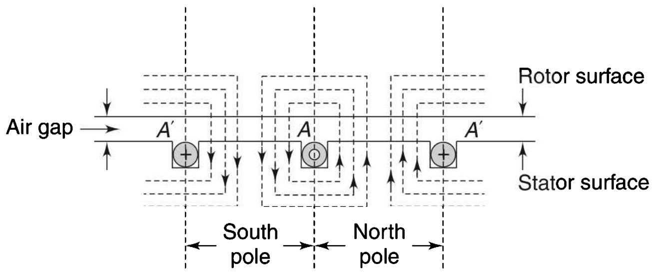
Let a stator of 2-pole machine have a full pitch coil with uniform air gap. A coil (AA′) has N turns and each turn carries a current i. The magnetic flux set up by coil is shown by dotted line as in Fig. 14.

The following assumptions are made for the distribution of coil mmf.

(i) The relative permeability of stator and rotor cores is infinite and reluctance is offered to the magnetic flux by the air gap only.

(ii) The air gap flux is radial.

The gap length is small with respect to the rotor diameter and the flux density is constant.



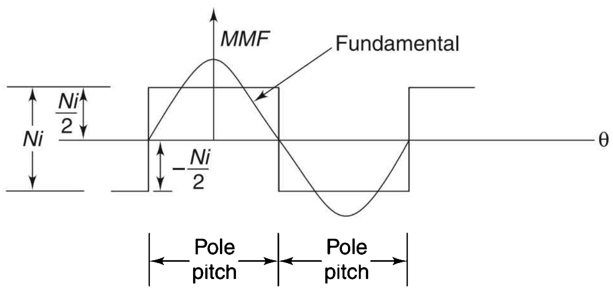


Fig. 14 MMF distribution for concentrated winding

Each flux line crosses the air gap twice. The mmf for any closed path is Ni. Now, reluctance of the iron is neglected, half the mmf $\frac{Ni}{2}$ is used to set up the flux from the rotor to the stator and the other half is used to set up the flux from stator to the rotor in the air gap. So, mmf of each air gap is $Ni/2$. The air gap mmf on the opposite side of the rotor is equal in magnitude but opposite in direction. If mmf coming out from the rotor to the stator is assumed positive, then from stator to rotor is negative. Figure 9.14 shows the air gap mmf distribution which is rectangular. Now the coil has a narrow space on the stator so mmf changes suddenly from positive to negative at one slot and in the reverse direction at the other slot.

The rectangular wave can be resolved into a Fourier series. It contains a fundamental component and series of odd harmonics. By Fourier series, the fundamental component is

$$F_{a1} = \frac{4}{\pi}\frac{Ni}{2}\cos\theta$$

where θ, the electrical angle, is measured from the magnetic axis of the coil which coincides with the positive peak of the fundamental wave shown in Fig. 14. It is a sinusoidal space wave of peak value.

$$F_{1\,\text{peak}} = \frac{4}{\pi}\frac{Ni}{2}$$

Remember!

- The variation of mmf along the air gap periphery is of rectangular wave form and of magnitude $\frac{Ni}{2}$.
- The amplitude of the mmf varies with time, but not with space.
- The air gap mmf wave is time variant but space invariant.
- The air gap mmf wave at instant is rectangular.

13 MMF OF DISTRIBUTED WINDING

Let us consider a 3-phase, 2-pole machine. The winding of three phases are identical and are place 120 electrical degrees apart. The mmf wave is a series of steps of height $2\pi n i$ equal to the one ampere-turn in the slot where $\boldsymbol{i}$ is the coil current. The distribution winding produces almost close to sine wave as in Fig. ´ .15.

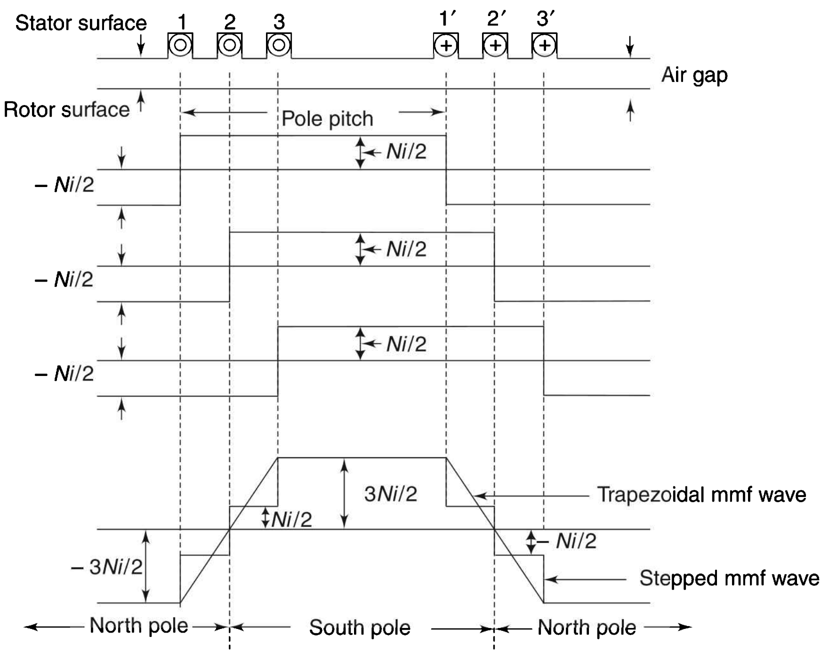


Fig. 15 MMF for distributed winding

- The mmf distribution along the air gap depends on the nature of slots, winding and exciting current.
- The shape of the mmf wave changes from rectangular to stepped, due to the effect of winding distribution.

. .14 MMF OF THREE-PHASE WINDINGS (ROTATING MAGNETIC FIELD)

Please refer to Chapter.1, Sec. .3.

15 EMF WAVEFORM FOR CONCENTRATED WINDING

We have seen that mmf waveform produced in the synchronous alternator in concentrated and distributed winding. Now we will study which type of emf is produced in the synchronous machine when its field winding is excited. Consider a 3-phase alternator of 3-phase winding in stator. For the ease of understanding and simplification, only one phase is considered. All the sides of the coil of this phase winding are arranged in the same slot, i.e., the winding is concentrated.

The induced emfs in all the sides are in phase and since these coils are in series, these emfs can be added arithmetically.

Emf induced in coil 1 = e_1,

Emf induced in coil 2 = e_2,

Emf induced in coil 3 = e_3,

Total emf $e = e_1 + e_2 + e_3$. The phasor diagram is shown in Fig. 16.

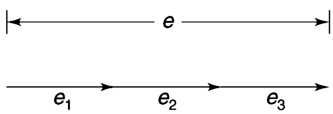


Fig. .16 EMF for concentrated winding

Considering the radial flux, the emfs will have flat topped waveform. The waveform of emfs is shown in Fig. 17.

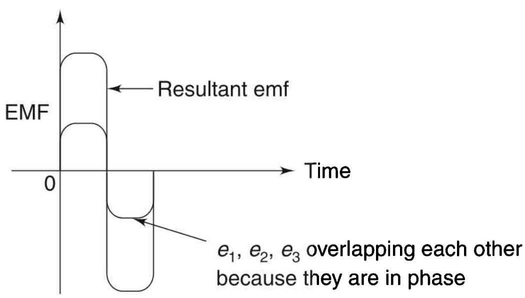


Fig. 17

Concentrated winding is not used in practice but distributed winding with short-pitch winding coil is used because of following drawbacks.

- It is not possible to get pure sine wave.
- More conductor length is required.
- Dissipation of heat is poor due to crowding of winding at one place.

16 EMF WAVEFORM FOR DISTRIBUTED WINDING

In this winding, coils are placed in different slots at an angle of $\alpha°$. The induced emfs, e_1, e_2, e_3 will now have a phase difference of $\alpha°$. Coil 2 will cut flux after coil 1 (after $\alpha°$.) and coil 3 will cut flux after coil 2 (after $\alpha°$.)

The total emf is obtained by phasor addition as in Fig. 18.

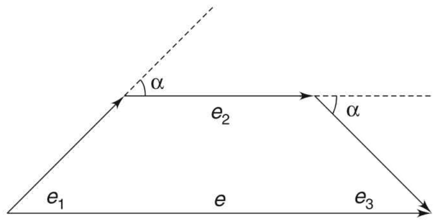


Fig. 18 Phasor diagram

Thus total emf, $e = \bar{e}_1 + \bar{e}_2 + \bar{e}_3$

The total emf induced in this winding is less compared to that in concentrated winding. The waveform is shown in Fig. 19.

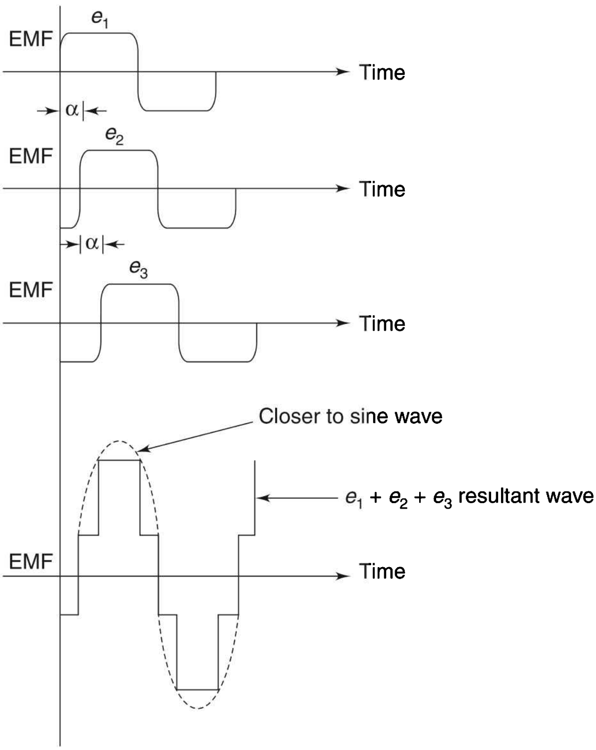


Fig. 19 EMF wave form for distributed winding

17 HARMONICS

Any undesirable components other than the fundamental are known as harmonics. They are found in each field of engineering. Vibrations and noise are the mechanical harmonics. Leakage and fringing are the magnetic harmonics while different waves produced at various frequencies are the electrical harmonics. We shall discuss only electrical and mechanical harmonics here. The main reason for their formation is the disturbance in the magnetic field. Harmonics form in generators and transformers due to following effect.

In any electrical rotating machine, it is expected that the flux produced by the pole should remain steady and uniform throughout the complete pole area. But in reality, this does not happen, because during magnetization and demagnetization, some dipoles are left in their original positions which generate various small poles of different polarities in different directions. These poles are known as residual poles. These residual poles produce flux and hence the voltage they produce is called harmonic voltage. Based on their frequencies, harmonics are termed as 2nd, 3rd, 4th, etc. They are further categorized as even harmonics (2nd, 4th, 6th, etc.) and odd harmonics (3rd, 5th, 7th, etc.)

Even harmonics are neutralized automatically because of their symmetry, i.e., their positive and negative cycles are same, while odd harmonics remain in the supply, disturbing the performance of the machine greatly, especially, the 3rd, 5th and 7th.

17.1 Effects of Harmonics

- The flux distribution along the air gap is non-sinusoidal so that the voltage generated in the individual conductor is also non-sinusoidal.
- According to Fourier, any periodic wave can be expressed as the sum of DC component (zero frequency) and sine wave or (cosine) waves having fundamental and multiple of higher frequencies called harmonics.

  The emf of a phase due to a fundamental component of the flux per pole is:

  $E_{ph1} = 4.44\, f k_{w1} T_{ph}\, \phi_1$, where $k_{w1} = k_{p1} k_{d1}$ is the winding factor.

  For the nth harmonic, $E_{\text{phn}} = 4.44 n\, f\, k_{wn} T_{ph}\, \phi n$.

  The nth harmonic and fundamental emf components are related by

$$\frac{E_{phn}}{E_{ph1}} = \frac{B_n K_{wn}}{B_1 K_{w1}}$$

  The RMS value of phase emf is $E_{ph} = \sqrt{(E_{ph1}^2 + E_{ph3}^2 + \dots + E_{phn}^2)}$

- All the odd harmonics (third, fifth, seventh, ninth, etc...) are present in the phase voltage up to some level and at the designing stage of the machine. This is important to note.
- Because the resulting voltage waveform is symmetric about the centre of the rotor flux, even harmonics are not present in the phase voltage.
- In Y connection, the third harmonic voltage between any two terminals will be zero. This result is also applied to any multiple of a third harmonic component (such as ninth harmonic). Such special harmonic frequencies are called triplen harmonics.

- The pitch factor of the coil at the harmonic frequency is given by

$$k_{cN} = \cos\frac{N\alpha}{2}$$

- Power factor is reduced because the reactance increases.
- All copper and core losses increase and hence the efficiency is decreased.

18 METHODS FOR ELIMINATING HARMONICS

If we make field flux waveform sinusoidal, harmonics can be greatly reduced. So, the following methods are used for the same purpose:

(i) Use of slots per pole per phase

If multiple sinusoidal emfs have small phase difference, their resultant will be sinusoidal. This can be done by using distributing winding. This is shown in Fig. 20.

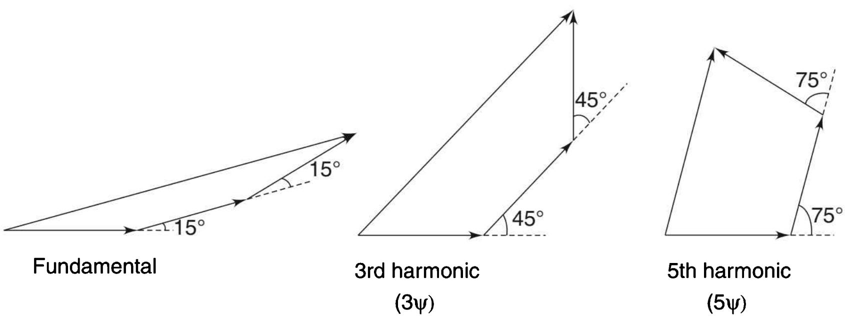


Fig. 20

Here, ψ is the slot pitch in electrical degrees of fundamental frequency.

(ii) Use of fractional pitch winding

In general, armature winding of the alternator is made with full number of slots per pole per phase. But, if we use fractional winding, i.e., $5\frac{1}{2}$ slots per pole per phase, the harmonics can be reduced.

In normal distribution winding, there is a small phase difference in voltages of the conductors of one group of coil but the next group occupies the same position in thereafter in same phase.

In fractional pitch winding, the coil groups do not occupy same position with respect to the pole. Hence, phase difference is produced between the groups of coils making one phase.

(iii) Use of short pitch winding

In this winding, coil width is less than the pole pitch. If this distance is $\frac{1}{n}$ th of the pole pitch, then nth harmonic will be removed because in the emfs of the two coil side it will be in opposition.

(iv) Skewing the pole faces

This is generally used for removing slot harmonics. So, first we need to know what slot harmonics are.

Use of distributed winding produces slot harmonics, or tooth ripple. The reluctance of the flux through the teeth is less than that through the slots which make deformations of the flux at the tip of the tooth. This happens because as the rotor poles move past the armature teeth, the configuration of the air gap changes. So, reluctance varies from teeth to slot as in Fig. 21.

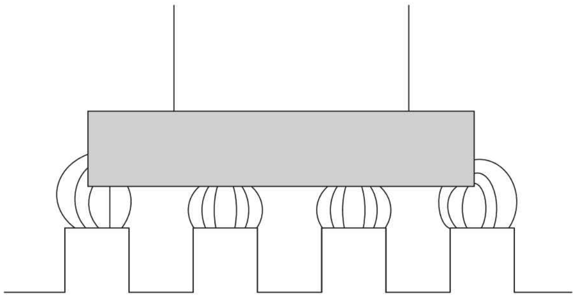


Fig. 21

These variations of reluctance set up an extra stationary wave of magnetism. This can be minimized by

- Use of fractional slot winding.
- Use of short pitch winding.
- Use of skewed rotor conductors.

In case of a 3-phase alternator, the rotor slots are not parallel to the shaft axis but have a small angular shift from shaft axis called skewing, as in Fig. 22. The other methods for eliminating the slot harmonics are:

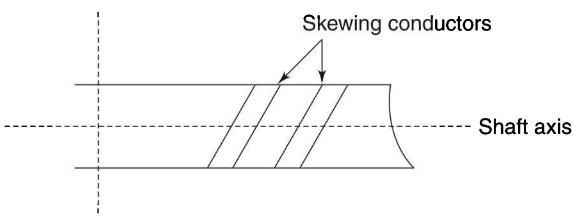


Fig. 22 Skewing of rotor conductors

(1) By using semi-closed or totally enclosed stator slots.
(2) By using shunt filter.
(3) By using multi-phase circuits
(4) By increasing the length of the air gap

19 ARMATURE REACTION IN SYNCHRONOUS GENERATORS

In DC machines, the effect of the armature reaction is that it distorts the air gap flux and changes its magnitude. The direction and magnitude depends on the position of the brushes. In the alternator, the direction and magnitude are not dependent on the position of the brushes but depend on phase displacement of the phase current and emf induced in the stator.

In an alternator, the phase angle ϕ can be within the limit of $\frac{-\pi}{2} \leq \phi \leq \frac{\pi}{2}$. So, we will consider three cases: (i) when $\phi = 0$, i.e., power factor is unity, (ii) when $\phi = -\frac{\pi}{2}$, i.e., the power factor is zero lagging, and (iii) when $\phi = \frac{\pi}{2}$, i.e., the power factor is zero leading.

When the current flows through the armature windings of the alternator, an mmf is produced which in turn produces the flux. The armature flux reacts with the main field and the resultant flux is either less than or more than the main pole flux.

Thus, the effect of armature flux on the flux produced by the rotor field poles is called armature reaction.

Consider a 3-phase, 2-pole machine having a single layer concentrated winding as in Fig. 23(a). Each phase has N number of turns. Let the rotor of the alternator be rotating in clockwise direction, i.e., the stator conductors are moving in anti-clockwise direction. Hence, by using the Fleming's right hand rule, the direction of the induced emf can be found. Now, suppose that the alternator is loaded and connected with resistive load. The currents that will flow in the three phases of the alternator are I_1, I_2 and I_3 and they are in phase with their respective phase voltages. The phasor diagram of phase voltage and phase current is shown in Fig. 23(b). The waveform of the current I_1, I_2 and I_3 is shown in Fig. 23(c). From the figure we see that at a time t_1, current I_1 will be maximum, and I_2, and I_3 have half their maximum values. Further, at time t_1, the current I_1 is positive and the other two currents I_2 and I_3 are negative.

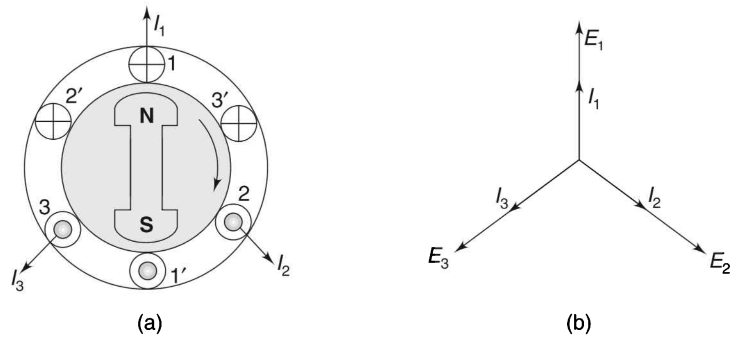


Fig. 23

From Fig. 27(c), the mmf produced by phase 1 – 1' is horizontal and its value is equal to $N\,I_{max}$. While the other two mmfs produced by two phases are at 60° to the horizontal and their value is $N\,\frac{I_{max}}{2}$. The total armature mmf is the vector sum of these emfs.

So, armature mmf = $N\,I_{max} + 2\left(N\,\dfrac{I_{max}}{2}\right)\cos 60° = 1.5\,NI_{max}$

At the instant t_1, the main field mmf is upwards and the armature mmf lags behind it by 90° electrical degrees in space. Note that the field mmf and armature mmf both are space vectors and both are rotating at synchronous speed. We can say that both are stationary with respect to each other and rotating at synchronous speed with respect to the stationary armature conductors.

Now we consider the other moment of time. At the instant $t_{2,}$ the poles are in horizontal position and $I_1 = 0$, but I_2 and I_3 are equal to 0.866 of their maximum value. Here, I_3 has not changed its direction hence the direction of mmf of I_3 is unchanged but the direction of I_2 has changed, hence its mmf will be in position as shown in Fig. ' 27(c). The total armature mmf is again the vector sum of these two mmfs.

Armature mmf = $2(0.866\,NI_{max})\cos 30° = 1.5\,NI_{max}$

If we proceed further to any instant of time, it is found that:

- The value of the armature mmf always remains constant with time.
- It is behind the main field mmf by 90° in space (not in time) so it consists of distorting nature.
- As said above, it rotates at synchronous speed round the stator.

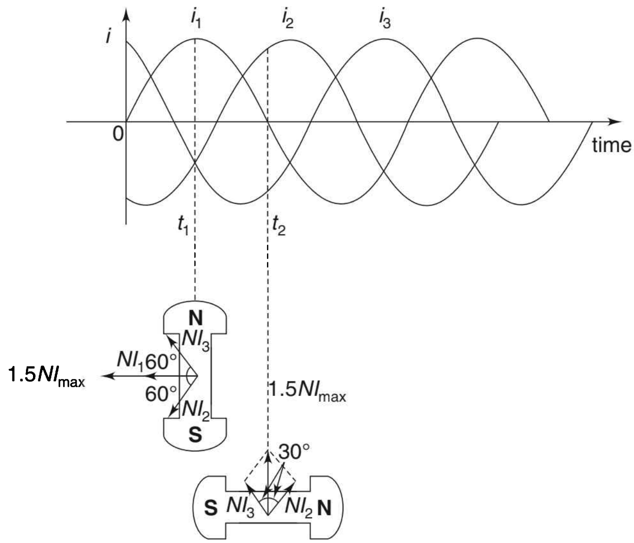


Fig. 23(c)

When the alternator is connected with lagging load of zero power factor, all the currents will be delayed in time by 90° and armature mmf will be shifted 90° as in Fig. 23(d) when compared with Fig. ⸝.23(c). Hence, the armature effect is purely demagnetizing, and reduces induced emf and terminal voltage.

When the alternator is connected with a leading load of zero power factor, the armature mmf strengthens the main mmf. Here, the armature reaction effect is wholly magnetizing, as it increases induced emf and hence terminal voltage as in Fig. .23(e).

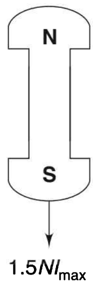


Fig. .23(d)

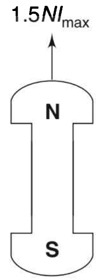
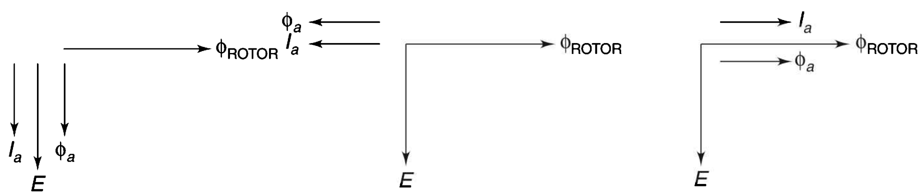


Fig. 23(e)

This effect of the armature reaction can also be understood by phasor as shown in Fig. ‘ 24(a).



Resistive load (unity power factor) Inductive load (zero p.f. lagging) Capacitive load (zero p.f. leading)

Fig. .24(a) Armature reaction for different load

The rotor field flux ϕ_{ROTOR}, produces induced emf, *E,* in the armature winding. When the generator is loaded, the armature current I_a flows in the armature winding and the load. This armature current produces a flux, ϕ_a, in the air gap.

The phase relationship between the induced emf *E* and the current flowing through the armature winding depends on the power factor of the load. This is shown in Fig. ‘ .24(a).

The armature reaction in synchronous machine is different for different power factors. It depends on the magnitude of the armature current as well as power factor of the load. Hence, we can summarize the effect of the armature reaction in the performance of the alternator as in Fig. .24(b).

Conclusion: From the above discussion, it can be easily concluded that the effect of the armature reaction depends on the load as well as on power factor of the load.

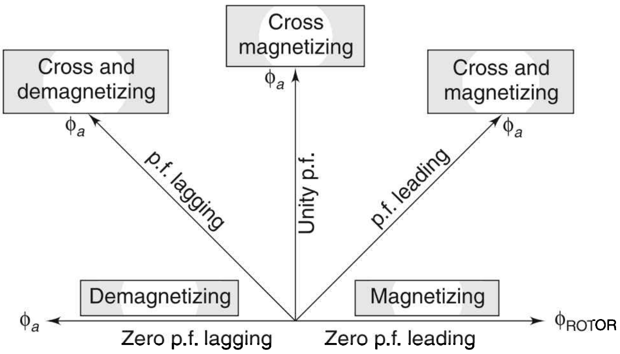


Fig. 24(b)

Remember!

- The armature reaction flux is rotating at synchronous speed and its magnitude is constant.
- It is cross-magnetizing when the generator is connected with a load of power factor of unity.
- When generator is connected with a load of lagging power factor, the armature reaction is purely demagnetizing.
- For a load of leading power factor, the armature reaction is magnetizing.
- In all cases, it is assumed that if the armature reaction flux is acted only, it produces voltage in each phase which lags behind the phase current by 90°.

19.1 Effect of Armature Reaction on Terminal Voltage

When the generator is loaded, the terminal voltage drops due to voltage drop in armature resistance, voltage drop due to armature leakage reactance and the effect of armature reaction. The load characteristic of the alternator at various power factors is shown in Fig. 25.

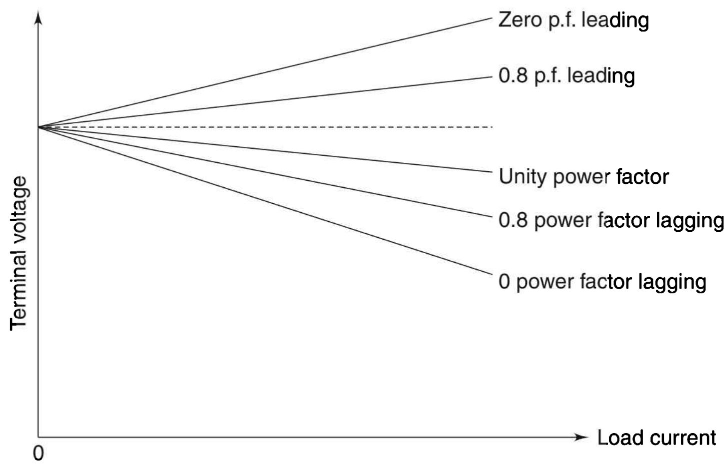


Fig. 25 Load characteristic of alternator for different power factor

19.2 Armature Leakage Reactance (X_L)

When the alternator is loaded, the terminal voltage drops due to the following factors:

- Armature resistance (R_a)
- Armature leakage reactance (X_L)
- Armature reaction reactance (X_a)

In the transformer, power is shared between two circuits via magnetic coupling. In the same way, in the rotating machine, power is transferred via magnetic coupling. So, the performance of the electrical machine is highly affected by the magnetic coupling between the two systems. So, in any electrical machine, some flux set up by one system does not link with the other system is known as a leakage flux or any flux which does not contribute to the useful flux is called as a leakage flux as in Fig. 9.26. The leakage flux may be

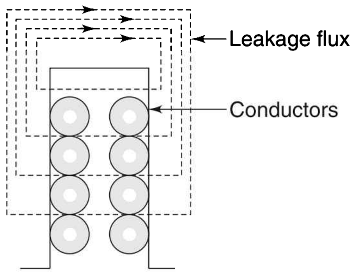


Fig. 26 Leakage flux

- Slot leakage.
- Tooth head leakage.
- Coil-end or overhang leakage.

This leakage flux affects the machine in the following ways:

(i) Less amount of power is transferred from one system to the other.
(ii) Less voltage is produced in case of alternator.
(iii) However, the power is not wasted but is stored in magnetic form.

All above properties are shown in the reactance. Hence, the effect of leakage flux is presented by the fictitious reactance called leakage reactance (X_L) connected in series with the circuit. This leakage flux produces self induced emf.

Most of the reluctance of magnetic circuits for armature leakage flux is due to air paths. The fluxes are nearly proportional to the current producing them and are in phase with them. So, the voltage induced by this flux in the armature winding is taken into account by the use of constant leakage reactance, multiplied by phase current. This voltage is leakage reactance drop and leads the current producing them by 90°.

19.3 Armature Reaction Reactance (X_a)

In the alternator, the main flux is produced by the rotor. When the alternator is loaded the armature current flows in the armature winding and the load. This armature current produces its own flux ϕ_a, which affects the main field flux either demagnetizing or cross-magnetizing it.

Due to the armature reaction, the voltage drops by X_a. This voltage is equivalent to the voltage of the inductive reactance.

This is a totally fictitious reactance which produces a voltage drop in the armature voltage. So, this voltage can be modeled in the same way as we connect an inductor in series with the internal generated voltage.

19.4 Synchronous Reactance (X_S)

The leakage reactance (X_L) totally depends on the frequency, because $X_L = 2\pi f L$. The alternator always operates at constant frequency or synchronous frequency. Hence, this reactance is called synchronous reactance. So, the leakage reactance (X_L) and reactance due to armature reaction effect (X_a) are combined together and this combination is called synchronous reactance (X_s)

$$X_s = X_L + X_a \quad (19.1)$$

The effective voltage drop due to synchronous reactance ($I_a X_s$) is shown leading the current by 90°. This is a totally fictitious reactance.

19.5 Synchronous Impedance (Z_s)

The above fictitious reactance when added with armature resistance R_a, in impedance triangle diagram, shown in Fig. 27, the addition is called synchronous impedance.

$$Z_s = R_a + j\,X_s \text{ or } Z_s = \sqrt{R_a^2 + X_s^2} \quad (19.2)$$

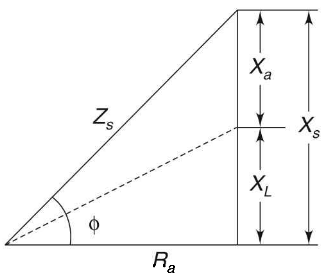


Fig. 27 Impedance triangle

> The synchronous reactance and synchronous impedance both are fictitious.

20 EQUIVALENT CIRCUIT AND PHASOR DIAGRAM OF THE SYNCHRONOUS GENERATOR

Equivalent circuit of a synchronous alternator is shown in Fig. 28(a) which can be redrawn from Fig. 28(b) by taking $X_s = X_L + X_a$.

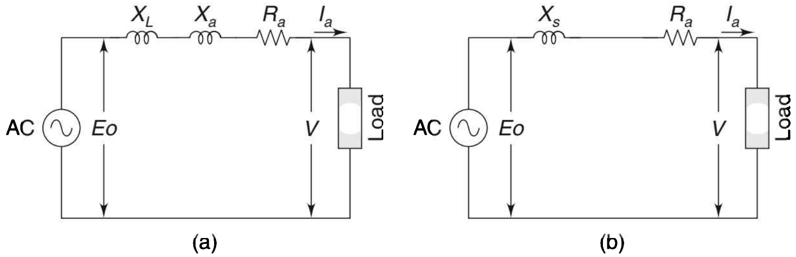


Fig. 28 Equivalent circuit of alternator

20.1 Lagging Power Factor

The phasor diagram for lagging power factor is shown in Fig. 29. In this diagram the terminal voltage V is taken as reference phasor such that OP = V. For lagging power factor cos ϕ, the direction of the armature current I_a, lags behind the voltage V by an angle ϕ, where OQ = I_a. The voltage drop in the armature resistance (I_aR_a) is represented by phasor PR. The voltage drop across the synchronous reactance is I_aX_s which is represented by RS. It leads the current by 90° and therefore RS is drawn perpendicular to the OQ. The total voltage drop is the phasor sum of I_aR_a and I_aX_s, which is represented by PS. The phasor OS gives voltage E_0.

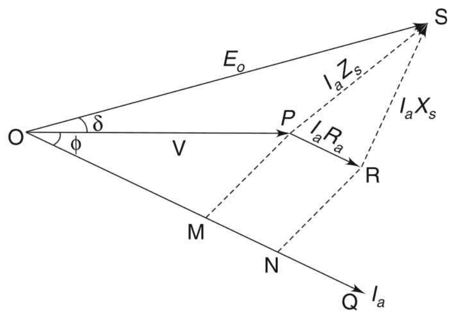


Fig. 29 Phasor diagram for lagging power factor

The magnitude of E_0 can be found from the triangle ONS.

$$OS^2 = ON^2 + NS^2 = (OM + MN)^2 + (NR + RS)^2$$

$$E_0^2 = (V \cos \phi + I_aR_a)^2 + (V \sin \phi + I_aX_s)^2$$

$$E_0 = \sqrt{(V \cos \phi + I_aR_a)^2 + (V \sin \phi + I_aX_s)^2}$$

The angle between E_0 and V is called the power angle or torque angle of the machine. It varies with the load and is a measure of air gap power developed in the machine.

20.2 Unity Power Factor

The phasor diagram for unity power factor is shown in Fig. 30.

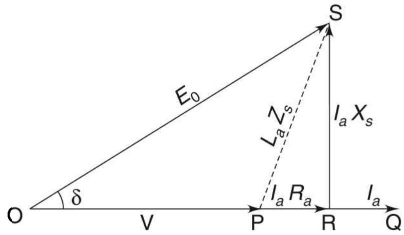


Fig. 30 Phasor diagram for unity power factor

From the right-angled triangle, ORS

$$(OS)^2 = (OR)^2 + (RS)^2 = (OP + PR)^2 + (RS)^2$$

$$E_0^2 = (V + I_aR_a)^2 + (I_aX_s)^2$$

$$E_0 = \sqrt{(V + I_aR_a)^2 + (I_aX_s)^2}$$

20.3 Leading Power Factor

The phasor diagram is shown in Fig. 31.

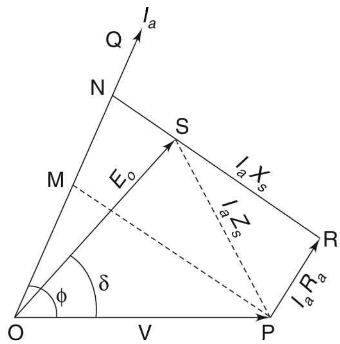


Fig. 31 Phasor diagram for leading power factor

From the right-angled triangle, ONS

$$OS^2 = ON^2 + NS^2 = (OM + MN)^2 + (NR - RS)^2$$

$$E_o^2 = (V \cos \phi + I_aR_a)^2 + (V \sin \phi - I_aX_s)^2$$

$$E_o = \sqrt{(V \cos \phi + I_aR_a)^2 + (V \sin \phi - I_aX_s)^2}$$

,21 VOLTAGE REGULATION OF AN ALTERNATOR

Voltage regulation of an alternator is given as the increment in terminal voltage when the load is reduced from full load to zero or no load, keeping the field excitation and speed constant.

$$\text{Voltage regulation of an alternator} = \frac{E_0 - V}{V} \times 100$$

where E_0 = No load voltage and V = Terminal voltage at load.

As per the modern trend, an alternator is built with high voltage regulation. To avoid severity of short circuit current, the reactance of the alternator is increased. So the voltage drop is also increased resulting in high voltage regulation.

21.1 Methods of Calculating Voltage Regulation

The methods generally used to find voltage regulation of smooth cylindrical rotor type alternator are:

(a) Direct load test,

(b) Indirect methods.

21.1.1 Direct load test

In this method, the alternator is run at a synchronous speed with the help of the prime mover, and its terminal voltage is adjusted to its rated value V. Now the load is connected and varied until the ammeter reads the full load reading. All the readings are recorded. Then the load is removed and the field and speed are kept constant. The open circuit voltage, $E_{0,}$ or no load voltage is recorded. From the above readings, voltage regulation is worked out with the help of the equation above.

This method is known as direct loading and is suitable only for small machines of power rating less than 5 kVA. The voltage regulation obtained by this method gives accurate results.

The main disadvantage of this method is that large amount of power is wasted and hence high capacity machines cannot be tested due to economic reason. For large alternators, indirect methods are used to find the voltage regulation of cylindrical rotor type alternator. These are: (i) synchronous impedance (emf) method, (ii) mmf method, and (iii) ZPF method as discussed below.

21.1.2 Synchronous impedance method

This method is also known as emf method because in this method armature resistance drop I_aR_a, and armature leakage reactance drop I_aX_L are actually emf quantities. In this method, the effect of armature reaction is replaced by fictitious reactance.

We know that for the alternator, $V = E_0 - Z_sI_a$, where $Z_s = R_a + jX_s$

So the value of Z_s is measured and then E_0 and V, are calculated. From that voltage regulation is found. To know the value of synchronous impedance, the following three tests are performed on the alternator.

(i) DC resistance test

Keeping the field winding open, the resistance of alternator per phase is measured by ammeter-voltmeter or by Wheatstone's bridge. The alternator should be at rest. Now the value of the effective AC resistance is larger than DC resistance due to skin effect. So, the value of DC resistance is multiplied by a factor of 1.20 to 1.75 depending on the size of the machine. Generally, 1.25 is taken for calculation.

(ii) Open circuit test

The alternator is run at synchronous speed and armature terminals are kept open as in Fig. 32. Then the field current is gradually increased and the terminal voltage is measured. The field current is so increased to get 25 to 30% more than the rated voltage. The graph is drawn between open-circuit phase voltage and the field current. This characteristic is known as open-circuit characteristic (OCC). It is similar to magnetizing curve. The OCC curve is a straight line in the beginning as the magnetic circuit of the alternator is not saturated and most of the applied mmf is consumed in the air gap. After the saturation, the OCC curve start to bend from the air gap line as shown in Fig. 33.

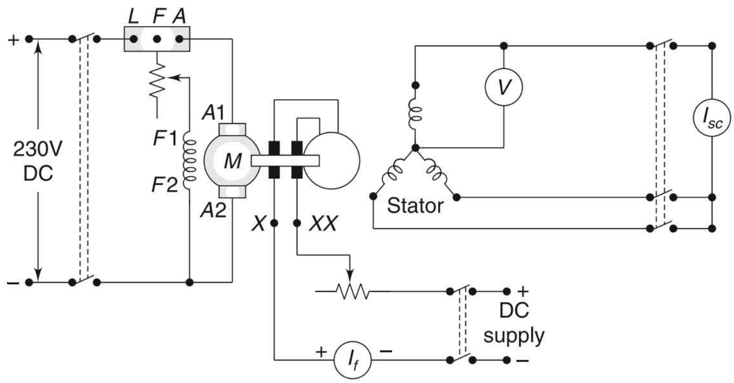


Fig. 32 Circuit diagram for OC and SC test

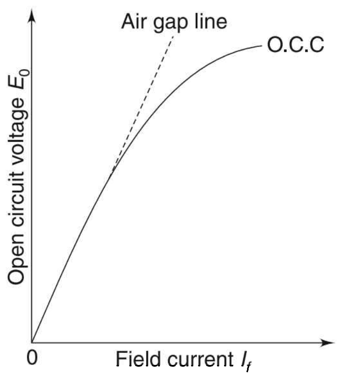


Fig. 33

(iii) Short circuit test

The armature terminals are shorted through ammeter as in circuit diagram by closing the switch. Then the alternator is run at the synchronous speed and field current is so adjusted that the ammeter gives the full load current. The graph is plotted between armature current

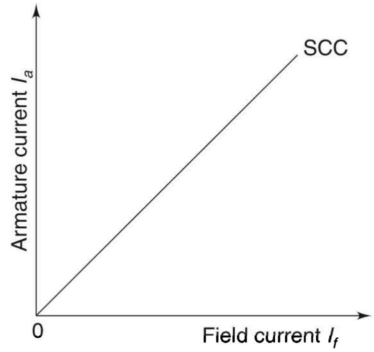


Fig. 34 SCC of alternator

and field current which is known as short-circuit characteristic (SCC). This gives a straight line as shown in Fig. 34.

Calculation of Z_s

Both OCC and SCC are drawn on the same graph. Find the value of I_{sc} at the field current that gives the rated voltage per phase of alternator. Now, synchronous impedance, Z_s is equal to the open-circuit voltage divided by the short-circuit current at that field current which gives the rated emf per phase.

$$Z_s = \frac{\text{Open-circuit voltage per phase}}{\text{Short-circuit armature current}} \text{ for the same value of field current}$$

Synchronous reactance is found as $X_s = \sqrt{Z_s^2 - R_a^2}$

From Fig. 35, the field current I_f = OP that produces the rated voltage per phase PQ. Moreover Z_s can be found for any load condition (Fig. 35).

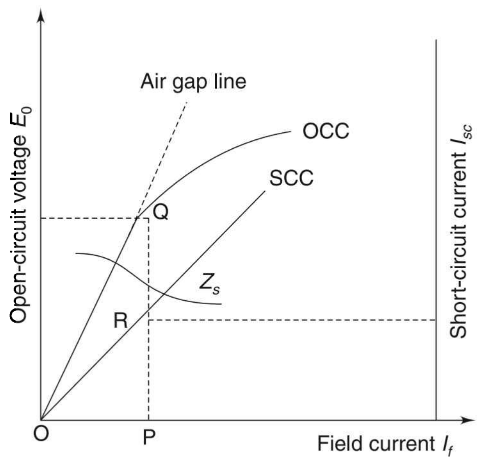


Fig. 35

Assumptions accepted in the emf method

(i) In a synchronous machine, the synchronous impedance is not constant all the time because when OCC and SCC are linear, the value of Z_s is constant but above the knee point of OCC when saturation starts its value is decreased as shown in graph. In the emf method, we do not consider the effect of saturation. This is the big error of this method.

(ii) In this test, it is assumed that the flux under whole the test is the same. But it is not true. When the armature is short circuited during the SC test, the armature current lags behind the voltage by almost 90° so armature reaction is completely demagnetizing and because of that the saturation is more reduced. Hence, the resultant flux and hence generated voltage is small. These conditions are different from the actual load condition. Hence, the OCC voltage found in this method is large. So the value of the synchronous impedance found by this method is too large.

(iii) In this method, the effect of armature reaction is replaced by a voltage drop proportional to the armature current. This voltage drop is added to the armature reactance voltage drop. This is not correct because armature flux vary with the power factor and the load current. So armature reaction voltage is not in phase with armature reactance drop.

(iv) We assumed that the reluctance offered to the magnetic circuit is constant. This is somehow true for cylindrical rotor type alternator because of having a uniform air gap. But in the salient pole machine, the position of armature flux relative to the field poles changes with the power factor. So, the variation in reluctance and armature flux with power factor introduces considerable error in salient pole machine.

So, considering above factors, we can say that the regulation found by this method is higher than actual loading. Hence this method is called pessimistic method also.

When the excitation is low, the value of synchronous impedance (Z_s) is constant which is called the linear or unsaturated synchronous impedance. But beyond the linear part of the OCC is called the saturated synchronous impedance. These values are not constant but vary with the excitation, that is, with the voltage, current and power factor. The value to be used in a given condition is called the effective synchronous impedance.

Unsaturated and saturated synchronous reactance

The unsaturated synchronous reactance X_{un} can be obtained from the air gap line voltage and the short-circuit current of the machine for a particular value of field current. From Fig. .36,

$$Z_{un} = \frac{PS}{PQ} = R_a + jX_{un} \text{ if } R_a \text{ is neglected,}$$

$$X_{un} = \frac{PS}{PQ}$$

We know that the synchronous reactance varies with degree of saturation of the OCC. So the value of synchronous reactance should be calculated at the load on the machine. If the machine is now connected to the infinite bus and its terminal voltage V, remains the same at

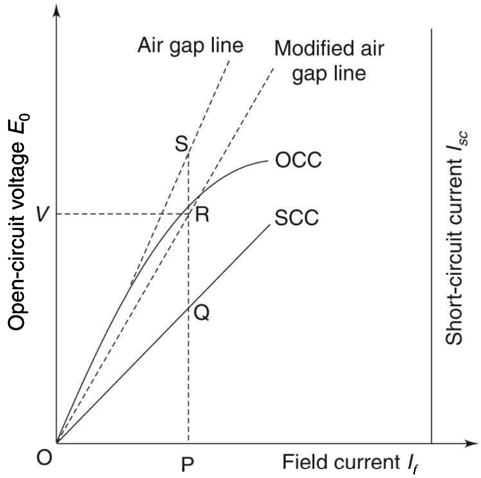


Fig. 36

the bus bar voltage. Now if the field current is changed, the voltage will change not on the OCC but on OC, called modified air gap line which represents the same magnetic saturation level at point R.

The saturated synchronous reactance at the rated voltage is found as follows:

$$Z_{sat} = \frac{E_{RP}}{I_{QP}} = R_a + jX_{sat} \text{ if } R_a \text{ is neglected, } X_{sat} = \frac{E_{RP}}{I_{QP}}$$

Example .8 A single-phase alternator is producing the 210 V in OC test and produces 200 A armature current on short circuit. If the effective armature resistance is 0.15 Ω, find (i) synchronous impedance, (ii) synchronous reactance.

Solution: Synchronous impedance, $Z_s = \frac{\text{OC voltage}}{\text{SC current}} = \frac{210}{200} = 1.05\ \Omega.$

Synchronous reactance, $X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{1.05^2 - 0.15^2} = 1.03\ \Omega$

Example .9 A 3-phase, 100 kVA, 3300 V, star connected alternator has effective resistance of 0.25 Ω. A 50 A field current produce a 200 A short-circuit current and 1050 OC line voltage. Find the voltage regulation at (i) unity (ii) 0.8 lagging and (iii) 0.8 power factor leading.

Solution: OC line voltage = 1050 V, so phase voltage = $\frac{1050}{\sqrt{3}} = 606$ V

Full load rated current = $\frac{100 \times 1000}{\sqrt{3} \times 3300} = 17.5$ A

Synchronous impedance, $Z_z = \frac{\text{OC voltage}}{\text{SC current}} = \frac{606}{200} = 3.03\ \Omega$: $R_a = 0.25\ \Omega$

Synchronous reactance, $X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{3.03^2 - 0.25^2} = 3.01\ \Omega$

$$I_a R_a = 17.5 \times 0.25 = 4.37 \text{ V} \quad \text{and} \quad I_a X_s = 17.5 \times 3.01 = 52.67 \text{ V}$$

Rated voltage per phase, $V_p = \dfrac{3300}{\sqrt{3}} = 1905$ V

$$\cos\phi = 0.8; \quad \text{and} \quad \sin\phi = 0.6$$

(i) *Regulation at 0.8 power factor lagging:*

$$E_o = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_s)^2}$$

$$= \sqrt{(1905 \times 0.8 + 4.37)^2 + (1905 \times 0.6 + 52.67)^2} = \sqrt{1528.37^2 + 1195.67^2} = 1940 \text{ V}$$

$$\text{Voltage regulation} = \frac{E_o - V}{V} \times 100 = \frac{1940 - 1905}{1905} \times 100 = 1.8\%$$

(ii) *Regulation at 0.8 power factor leading:*

$$E_o = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi - I_a X_s)^2}$$

$$= \sqrt{(1905 \times 0.8 + 4.37)^2 + (1905 \times 0.6 - 52.67)^2} = \sqrt{1528.37^2 + 1090.33^2} = 1877 \text{ V}$$

$$\text{Voltage regulation} = \frac{E_o - V}{E_o} \times 100 = \frac{1877 - 1905}{1905} \times 100 = -1.46\%$$

(iii) *Regulation at unity power factor:*

$$E_o = \sqrt{(V + I_a R_a)^2 + (I_a X_s)^2} = \sqrt{(1905 + 4.37)^2 + 52.67^2} = 1910 \text{ V}$$

$$\text{Voltage regulation} = \frac{E_o - V}{E_o} \times 100 = \frac{1910 - 1905}{1905} \times 100 = 0.26\%$$

Example .10 A 3-phase, star-connected alternator is supplying a current at 0.8 power factor lagging. The no load terminal voltage is 4000 V and at full load of 2500 kW, it is 3300 V. Determine the terminal voltage when the alternator is delivering a current to a 3-phase star-connected load having a resistance of 10 Ω and reactance of 8 Ω per phase. Speed and field excitation are constant.

Solution: 3-phase power, $P_{3\phi} = 3V_p I_p \cos\phi$

Full load rated current, $I_p = \dfrac{2500 \times 1000}{3 \times (3300/\sqrt{3}) \times 0.8} = 546.80$ A

No load phase voltage, $= \dfrac{4000}{\sqrt{3}} = 2309$ V

Full load phase voltage, $= \dfrac{3300}{\sqrt{3}} = 1905$ V

Voltage drop per phase for a current of 546.80 A = 2309 – 1905 = 404 V

∴ Voltage drop for 1 A current = $\frac{404}{546.80} = 0.73$ V

Now the alternator is delivering the current I.

∴ Voltage drop per phase for supplying current I at 0.8 power factor lagging = $0.73I$ volts

∴ Terminal voltage per phase for supplying current I at 0.8 pf = $2309 - 0.73I$ volts

Load impedance, $Z_L = \sqrt{10^2 + 8^2} = 12.80\ \Omega$

Load terminal voltage, $IZ_L = I \times 12.80$

$$12.80\,I = 2309 - 0.73\,I$$

$$I = \frac{2309}{13.53} = 170.65 \text{ A}$$

Terminal voltage per phase, $IZ_L = 170.65 \times 12.80 = 2184$ V

Line voltage = $\sqrt{3} \times 2184 = 3782.80$ V

Example .11 A 3-phase, star-connected, 25 kVA, 415 V alternator has an armature resistance per phase and reactance per phase is 0.6 Ω and 1.4 Ω. Find the % voltage regulation at full load at power factor (i) 0.8 lagging, (ii) 0.8 leading. Also find the value of power factor when the regulation at full load is zero.

Solution:

$$S_{3\phi} = \sqrt{3}V_L I_L$$

$$I_L = \frac{25000}{\sqrt{3} \times 415} = 34.78 \text{ A} = I_{ap}$$

Rated voltage per phase, $V_p = \frac{415}{\sqrt{3}} = 239.60$ V

Let V_p is taken as reference phasor, $V_p \angle 0° = 239.60 \angle 0° = 239.60 + j0$

$$Z_s = R_a + jX_s = 0.6 + j1.4 = 1.52 \angle 66.82°\ \Omega$$

(i) *Power factor 0.8 lagging:*

$$I_{ap} = I_{ap} \angle -\cos^{-1} 0.8 = 34.78 \angle -36.86° \text{A}$$

$$E_p = V_p + I_{ap} Z_s = 239.60 + j0 + (34.78 \angle -36.86°) \times (1.52 \angle 66.80°)$$

$$= 239.60 + j0 + 52.86 \angle 29.94° = 239.60 + j0 + 45.80 + j26.38$$

$$= 265.98 + j26.38$$

$$= 267.28 \angle 5.66 \text{ V}$$

Voltage regulation = $\frac{E_p - V_p}{V_p} \times 100 = \frac{267.28 - 239.60}{239.60} \times 100 = 11.55\%$

(ii) *0.8 power factor leading:*

$$I_{ap} = I_{ap} \angle \cos^{-1} 0.8 = 34.78 \angle 36.86° \text{ A}$$

$$E_p = V_p + I_{ap} Z_s = 239.60 + j0 + (34.78 \angle 36.86°) \times (1.52 \angle 66.80°)$$

$$= 239.60 + j0 + 52.86 \angle 103.66° = 239.60 + j0 - 12.48 + j51.36°$$

$$= 227.12 + j51.36° = 232.85 \angle 12.74° \text{ V}$$

$$\text{Voltage regulation} = \frac{E_p - V_p}{V_p} \times 100 = \frac{232.85 - 239.60}{239.60} \times 100 = -2.81\%$$

(iii) Let ϕ is the required power factor angle

$$I_{ap} = I_{ap} \angle\phi = 34.78 \angle\phi \text{ A}$$

$$E_P = V_P + I_{ap}Z_s = 239.60 + j0 + (34.78 \angle\phi°) \times (1.52 \angle 66.80°)$$

$$= V_P + I_{ap}Z_s = 239.60 + j0 + 52.86 \angle\phi + 66.80°$$

$$= 239.60 + 52.86 \cos(\phi + 66.80°) + j52.86 \sin(\phi + 66.80°)$$

$$E_p^2 = [239.60 + 52.86 \cos(\phi + 66.80°)]^2 + [52.86 \sin(\phi + 66.80°)]^2$$

For zero regulation, $E_p = V_p = 239.60$ V

$$239.60^2 = [239.60 + 52.86 \cos(\phi + 66.80°)]^2 + [52.86 \sin(\phi + 66.80°)]^2$$

$$239.60^2 = 239.60^2 + 2 \times 239.60 \times 52.86 \cos(\phi + 66.80°) + 52.86^2 \cos^2(\phi + 66.80°) + 52.86^2 \sin^2(\phi + 66.80°)$$

$$= 239.60^2 + 2 \times 239.60 \times 52.86 \cos(\phi + 66.80°) + 52.86^2 \cos^2(\phi + 66.80°) + 52.86^2$$

$\therefore$ $$\cos(\phi + 66.80°) = \frac{-52.86}{2 \times 239.60} = -0.11 = \cos 99$$

$\therefore$ $$\phi = 99° - 66.80° = 32.2$$

Hence, $\cos\phi = 0.84$ leading

Example .12 A 3-phase, star-connected, 20 kVA, 1100 V, 50 Hz, alternator delivering the rated load at 0.8 power factor lagging. If the armature resistance is 0.55 Ω per phase and the synchronous reactance is 12 Ω per phase, find the torque angle and voltage regulation.

Solution: $S_{3\phi} = \sqrt{3} V_L I_L$

$\therefore$ $$I_L = \frac{20 \times 1000}{\sqrt{3} \times 1100} = 13.12 \text{ A} = I_{ap}$$

$$Z_s = R_a + jX_s = 0.55 + j12 = 12.01 \angle 87.37°$$

Rated phase voltage, $$V_p = \frac{1100}{\sqrt{3}} = 635 \text{ V}$$

Taking V_p is the reference phasor.

$$V_p \angle 0° = 635 \angle 0° = 635 + j0 \text{ V}$$

When power factor is 0.8 lagging:

$$I_{ap} = I_{ap} \angle -\cos^{-1} 0.8 = 13.12 \angle -36.86° \text{ A}$$

$$E_p = V_p + I_{ap}Z_s = 635 + j0 \text{ V} + (13.12 \angle -36.86°) \times (12.01 \angle 87.37°)$$

$$= 635 + j0 \text{ V} + 157.57 \angle 50.51° = 635 + j0 \text{ V} + 100.20 + j121.60$$

$$= 735.20 + j121.60 = 745.18 \angle 9.39°$$

$$E_p = 745 \text{ V} \quad \text{and} \quad \delta = 9.39°$$

$$\text{Voltage regulation} = \frac{E_p - V_p}{V_p} \times 100 = \frac{745 - 635}{635} \times 100 = 17.32\%$$

Example .13 A 500 V, 50 kVA, 50 Hz, single-phase alternator has effective resistance of 0.25 Ω. A field current of 10 A produces armature current of 200 A on SC test and an emf of 450 V on OC test. Find the synchronous reactance and voltage regulation at full load with 0.8 power factor lagging.

Solution:

$$S_{1\phi} = VI_a$$

$$I_a = \frac{50 \times 1000}{500} = 100 \text{ A}$$

$$\cos\phi = 0.8 \quad \text{and} \quad \sin\phi = 0.6$$

$$\text{Synchronous impedance, } Z_s = \frac{\text{OC voltage}}{\text{SC current}} = \frac{450}{200} = 2.25 \ \Omega$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{2.25^2 - 0.25^2} = 2.23 \ \Omega$$

$$E_o = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi + I_aX_s)^2}$$

$$= \sqrt{(500 \times 0.8 + 100 \times 0.25)^2 + (500 \times 0.6 + 100 \times 2.23)^2}$$

$$= \sqrt{425^2 + 523^2} = 673.90 \text{ V}$$

$$\text{Voltage regulation} = \frac{E_0 - V}{V} \times 100 = \frac{673.9 - 500}{500} \times 100 = 34.78\%$$

21.1.3 Magnetomotive force (mmf) method

In this method of regulation, the armature leakage reactance is replaced by an equivalent additional armature reaction mmf. The following data is required for the mmf method to find the regulation.

(i) Armature resistance per phase.
(ii) OCC characteristic of the alternator.
(iii) SCC of the alternator.

Since the rotor flux produces the emf in the stator or armature, this emf can be represented by field mmf or field ampere turns. The turns of the field are constant, so it is given by field current only. From the above characteristic I_{f1} is the field current which gives rated voltage on load and current I_{f2} which produces full load current in the SC test.

If we draw the equivalent circuit of the alternator while it is shorted as in Fig. 9.37, the current I_{f2} balances the impedance drop and the armature reaction drops on full load. Now, R_a is very small and X_L is also very small because of low voltage on short circuit, so impedance can be neglected. Hence, power factor on short circuit is almost zero lagging and the I_{f2} is used to overcome the armature reaction effect only and armature reaction is purely demagnetizing at full load.

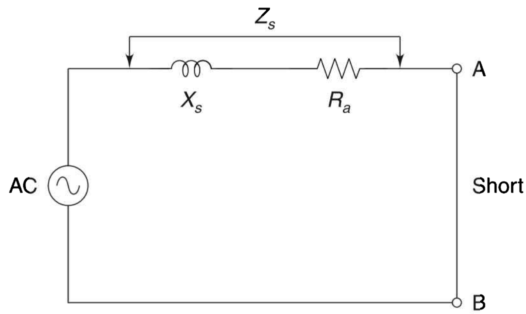


Fig. 37 Equivalent circuit when alternator is shorted

If the alternator supplies a full load current at zero power factor lagging, the total field AT required is the vector sum of:

(1) Field AT = I_{f1} = OF required to produce the normal voltage can be obtained from OCC.

(2) Field turn AT = I_{f2} = FF_1 required to neutralize the effect of armature reaction.

The total field ampere turn required is the vector sum of OF and FF_1. If the power factor is zero leading, the armature reaction is purely magnetizing. Hence the field ampere turns required is OF_2. It the pf is unity, the armature reaction is cross magnetizing; hence OF_3 is the vector sum of OF and OF_3 as shown in Fig. 9.38.

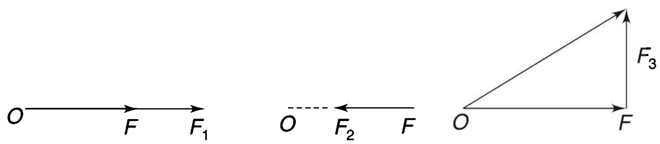


Fig. 38

Now, the alternator supplies full load current at power factor of cos ϕ, the field current I_{f1} to give full load voltage V (or more accurately, $V + I_a R_a \cos \phi$) is OF, then draw FF_1 at an angle (90° ± ϕ) as per Fig. 9.39 which represents the I_{f2} to give full load current, (+) sign is for lagging pf and (–) sign is for leading pf. Total field current I_f measuring OF_1 can be found which gives open-circuit emf E_0 which can be found from OCC. Now regulation of the alternator can be found by

$$\% \text{ Regulation} = \frac{E_o - V}{V} \times 100$$

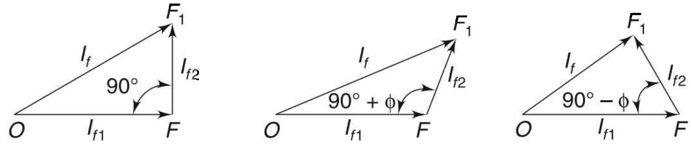


Fig. 39 Phasor diagram for mmf method

The complete diagram with OC and SC is shown in Fig. 40. Here, OF gives the field current for normal rated voltage. OC represents the field current required for producing full load current on SC. Vector $FF_1 = OC$ is drawn at an angle of $(90° + \phi)$ if the power factor is lagging. OF_1 is the total field current for which the corresponding voltage is E_0.

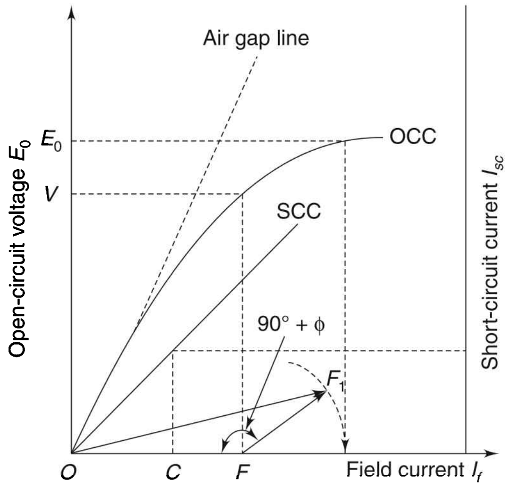


Fig. 40

This method is optimistic because it gives the value of regulation lower than the actual value. The reason is that the excitation to overcome the effect of armature reaction is determined on unsaturated part of the saturation curve.

Example .14 A 3-phase, star-connected, 1000 kVA, 2000 V, 50 Hz, gave following test results. The effective armature resistance per phase is 0.25 Ω. Draw the curve and find the full load regulation at 0.8 power factor lagging and leading.

Field current in A	10	20	25	30	40	50
OC voltage per phase	462	866	1016	1155	1357	1501
SC current in A	–	200	250	300	–	–

Solution: Full load phase voltage, $V_p = \dfrac{2000}{\sqrt{3}} = 1155$ V

Full load current, $$I_a = \frac{1000 \times 1000}{\sqrt{3} \times 2000} = 288.67 \text{ A}$$

Full load phase voltage at 0.8 power factor lagging = $V_p + I_a R_a \cos\phi$

$$= 1155 + 288.67 \times 0.25 \times 0.8 = 1201 \text{ V}$$

If OCC is drawn, the field current required to produce 1201 V is $I_{f1} = 32$ A

From SC, field current required to produced full load current is $I_{f2} = 29$ A

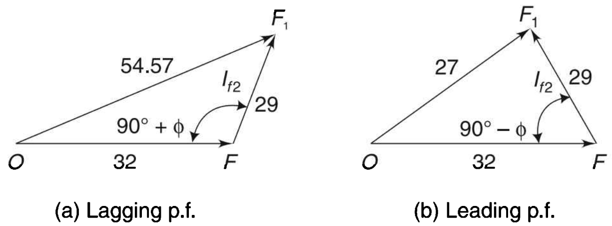


Fig .41

From Fig. .41(a),

OF = 32 A, FF_1 = 29 A and is at an angle of $(90° + \cos^{-1} 0.8) = 126.86°$

$$\therefore \quad OF_1 = \sqrt{32^2 + 29^2 + 2 \times 32 \times 29 \times \cos(180° - 53.14°)}$$

$$= \sqrt{1865 + 1856 \cos 53.14°} = \sqrt{2978} = 54.57 \text{ A}$$

OC voltage corresponding to this 54.57 A is 1555 V

$$\therefore \text{ Regulation} = \frac{1555 - 1155}{1155} \times 100 = 34.63\%$$

(ii) For 0.8 leading power factor: see Fig. 9.41(b)

Here, FF_1 is drawn at an angle of $(90° - \cos^{-1} 0.8) = 53.13°$

$$\therefore \quad OF_1 = \sqrt{32^2 + 29^2 + 2 \times 32 \times 29 \times \cos(180° - 126.87°)} = 27 \text{ A}$$

OC voltage corresponding to this 27 A is 1098 V

$$\therefore \text{ Regulation} = \frac{1098 - 1155}{1098} \times 100 = -4.93\%$$

.21.1.4 ZPF method

In all the methods of finding the regulation of alternator, E_0 is calculated and compared with V. Different voltage drops $I_aR_aI_aX_L$ are considered different as shown in the Table .1.

Table .1

Method	Consideration for voltage drop		
	$I_a R_a$	$I_a X_L$	$I_a X_a$
emf method	Considered as a part of emf	Considered as a part of emf	Considered as a part of emf
mmf method	Considered as a part of field mmf	Considered as a part of field mmf	Considered as a part of field mmf
zpf method	Considered as a part of emf	Considered as a part of emf	Considered as a part of mmf

Thus, in the zpf method, I_aR_a and I_aX_L are taken as emf quantity and I_aX_a means that the armature reaction drop is taken as mmf quantity. This is only the accurate assumption because I_aR_a, I_aX_L are voltage drop so they have to be taken as emf quantity and armature reaction is effect of armature flux on field flux so it must be taken as mmf quantity. So, $I_a X_L$ and $I_a X_a$ must be calculated separately.

For this method, OCC and zero power factor (zpf) tests are required to be performed.

ZPF test

Circuit diagram for the zpf test is shown in Fig. 42. The alternator is run at the synchronous speed. Then load is gradually increased till the full load current is achieved at rated terminal voltage and rated speed. As the load connected is inductive, the power factor is zero lagging, i.e., $\phi = 90°$ lagging. The field current is so adjusted that the ammeter current gives full load current.

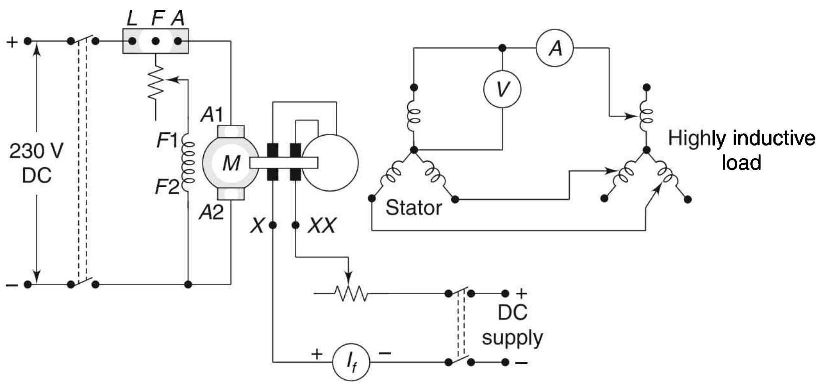


Fig. 42 Circuit diagram for zpf test

Now, the OCC is drawn. On the same graph, zpf curve is plotted. Now, point B is obtained when wattmeter gives zero reading. Point A is drawn from an SC test with full load current. Here, OA gives the value of field current which is equal to balance the demagnetizing armature reaction effect and for leakage reactance occurring drop at full load. By using these two points A and B, full load zpf curve is drawn. This is shown in Fig. 43.

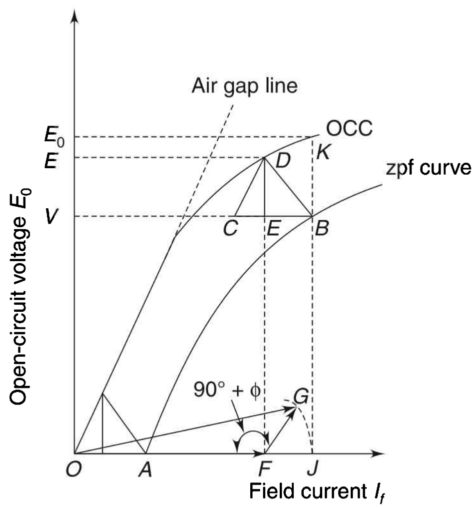


Fig. 43

From point B, line BC is drawn which is equal and parallel to OA. From point C, a line parallel to air gap line is drawn which cuts the OCC at point D. Thus, points B, C and D are obtained which make the ΔBCD. The ΔBCD is known as Potier triangle. For a given armature current, this triangle is constant and hence it can be moved on the zpf curve. A perpendicular DE is drawn perpendicular to the BC. The length

DE gives the voltage drop because of armature leakage reactance i.e. I_aX_L. BE gives field current required to balance demagnetizing effect of armature reaction and CE for balancing the armature leakage reactance drop DE.

If V is the full load terminal voltage and only armature leakage reactance voltage drop is added to it, we get E = DF. So, the field excitation is required to get E is OF. BE gives field current required to balance demagnetizing effect of armature reaction. So if we add BE vectorially to OF, we get excitation for obtaining E_0, whose value can be known from OCC. In Fig. ̗.47, lagging power factor is used. The voltage corresponding to this field excitation is JK = E_0.

$$\% \text{ Regulation} = \frac{E_o - V}{V}$$

Vector diagram for lagging power factor is given in Fig. 44. Terminal voltage V is drawn; also current I_a is drawn with an angle ϕ. I_aR_a drop is drawn parallel to the current I_a. Drop I_aX_a is drawn perpendicular to it. OA represents the voltage E. The value of field excitation corresponding to voltage E is OC and it is drawn at 90° ahead of it. Similarly, the other drops are added and finally we get the vector diagram. The angle δ between E_0 and V is the power angle or load angle or torque angle.

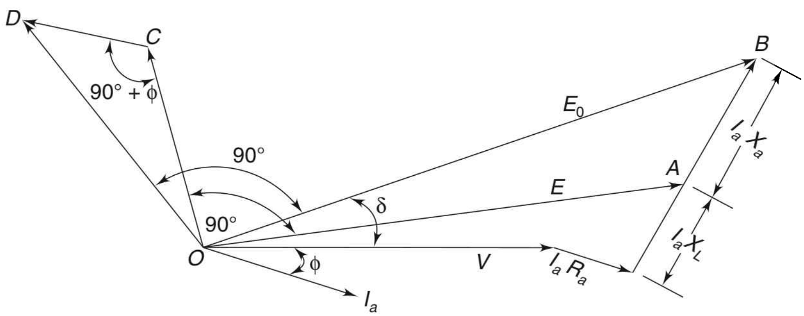


Fig. .44 Vector diagram for lagging power factor

> **Remember!**
> The emf induced due to electromagnetic induction lags the flux by 90° – Transformer basics.

21.2 AIEE Method for Voltage Regulation

In emf and zpf methods, it is assumed that the magnetic circuit of the alternator is unsaturated. However, this is not true and it affects the result of regulation. Secondly, field mmf and armature mmf cannot be considered as combined because each has a different magnetic circuit.

AIEE method which is also known as ASA (American Students Association) is used to find the regulation of alternator. This is a modification of the mmf method which is reliable and gives actual result for both salient-pole and non-salient pole machines.

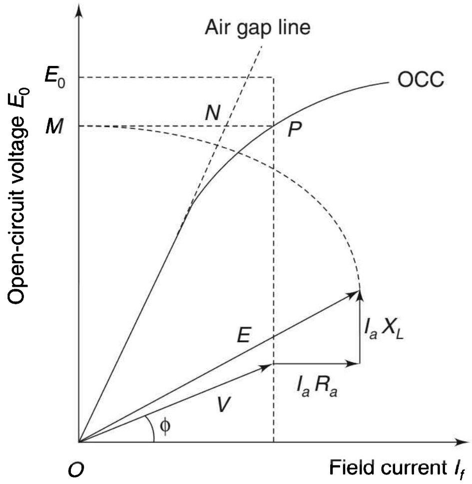


Fig. 45

In this method, saturation effect is considered. Here, OCC and air gap line are plotted as in Fig. 45. Taking the horizontal axis as a current, terminal voltage V is drawn at an angle ϕ. $I_a R_a$ drop is added at the tip of V and parallel to the current axis. Then $I_a X_L$ drop is added. The resultant of this drop gives the voltage E. Taking E as radius and O as center, an arc is drawn which cuts the Y-axis at point M. From point M, horizontal line is drawn which cuts air gap line at N and OCC at P as shown. The horizontal distance MP is the additional field current to take accountability of saturation effect. Now the mmf diagram can be constructed as before, considering lagging power factor.

I_{f1} = Field required to generate rated voltage on OCC.

I_{f2} = Field required to balance effect of armature reaction.

I_f = Resultant of above. To consider saturation effect, field current I_{f3} is added to I_f as in Fig. 46.

So, OA = $I_f + I_{f3'}$. Corresponding to this field current emf E is obtained from OCC and from that regulation can be found.

$$\% \text{ Regulation} = \frac{E_o - V}{V} \times 100$$

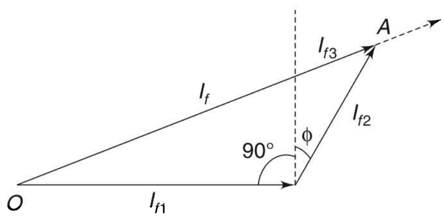


Fig. 46

Example .15 A 5000 kVA, 6600 V, 3-phase, star-connected alternator has a resistance of 0.075 Ω per phase. Find the voltage regulation for a load of 500 A at power factor unity, 0.9 leading and 0.71 lagging, by using the ZPF method. The test data are as under:

Field current A	OC voltage, V	ZPF curve, V
32	3100	0
50	4900	1850
75	6600	4250
100	7500	5800
140	8300	7000

Solution: The OCC and zpf curves are shown in Fig. 47

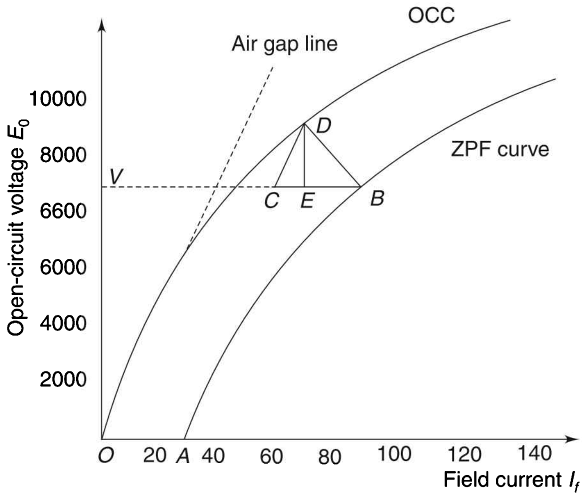


Fig. 47

Draw a horizontal line at rated voltage of 6600 V which cuts the ZPF at point *B*. On this line draw BC = OA = 32 A. OA is the field current required to circulate full load current on SC test. Draw a line CD parallel to air gap line which meets the OCC at point *D*. Draw a perpendicular DE on line BC. Hence, we get the Potier's triangle.

Now, BE is the field current required to balance the armature reaction and its value is of 25 A (measurement from the graph). The length DE gives the voltage drop due to the armature reactance, i.e., $I_a X_L$ and its value is measured from the OCC which gives the 900 V (line).

$\therefore$ $I_a X_L$ = 900 V so, per phase armature leakage reactance drop is $\frac{900}{\sqrt{3}} = 519.61$ V

$$\therefore \quad X_L = \frac{519.61}{I_a} = \frac{519.61}{500} = 1.039\ \Omega$$

Taking I_a as reference phasor, $I_a \angle 0° = 500 + j0$

(i) *Unity power factor:*

$$V_P = V_P \angle 0° = \frac{6600}{\sqrt{3}} = 3810.51 \text{ V}$$

$$E = V_P + I_a Z_L = V_P + I_a(R_a + jX_L) = V_P + I_a R_a + J I_a X_L$$

$$= 3810.51 + 500 \times 0.075 + j\,519.5 = 3848 + j\,519.5 = 3882.90 \angle 7.68° \text{ V}$$

$$E_L = \sqrt{3} \times 3882 = 6725 \text{ V}$$

From the OCC, field current required at 6725 V is 78 A which is given by OF (I_r) and is lead E by 90 as in Fig. ` 48.

∴ $$I_r = I_r \angle 90° + 7.69° = 78 \angle 97.69° \text{ A}$$

FG is drawn parallel to load current which gives the field current equivalent to full load armature reaction current (I_{ar}) OF is the total field current (I_f)

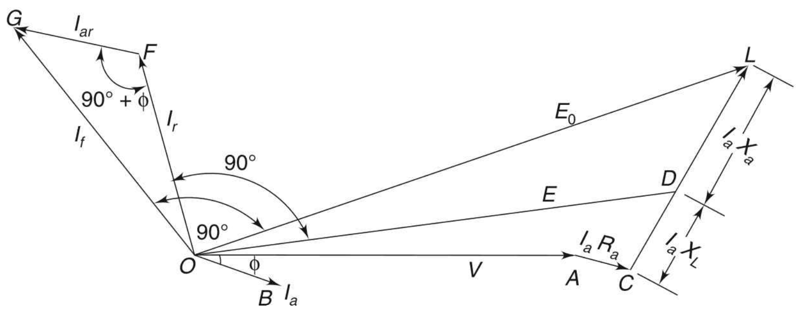


Fig. 48 Vector diagram for lagging power factor

The current I_{ar} is in phase with I_a. Hence, $I_{ar} = I_{ar} \angle 0° = 25 \angle 0°$

We have $I_f + I_{ar} = I_r$

∴ $I_f = 78 \angle 97.69° - 25 \angle 0° = -10.43 + j77.29 - 25 = -35.43 + j77.29 = 85 \angle 114.6$

From the OCC, voltage (E_0) corresponding to this current (85) is 7000 V (line).

$$E_{0p} = \frac{7000}{\sqrt{3}} = 4041.45 \text{ V}$$

∴ Voltage regulation $= \dfrac{E_{0p} - V_p}{V_p} = \dfrac{4041.45 - 3810.51}{3810.51} \times 100 = 6.06\%$

Alternate Method

From Fig. 48, OA = 3810.51 V, AC = 37.5 V, CD = 519.61 V

∴ $$\text{OD} = E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_L)^2}$$

$$= \sqrt{(3810.51 + 37.5)^2 + 519.61^2} = 3882 \text{ V}$$

$$E_L = \sqrt{3} \times 3882 = 6725 \text{ V}$$

From the OCC, field current required at 6725 V is 78 A. Hence, OF = 78 A, FG = 25 A and it is drawn at an angle of (90° + ϕ).

$OG = \sqrt{OF^2 + FG^2 + 2 \times FG \times OF \times \cos(90° + \phi)}$

$= \sqrt{78^2 + 25^2 + 2 \times 78 \times 25 \times \cos(\pi - (90° + \phi))} = \sqrt{6709 + 3900 \cos(90 - \phi)}$

$= \sqrt{6729 + 3900 \times \sin\phi} = 82$ A

(ii) 0.9 *power factor leading*

$\cos\phi = 0.9$. Hence, $\phi = 25.84°$ and $I_a \angle 0°$

$V_p = 3810.51 \angle -25.84°$ V $= 3429.51 - j1660.84$ V

$E = V_p + I_a Z_L = V_p + I_a (R_a + jX_L) = V_p + I_a R_a + jI_a X_L$

$= 3429.51 - j1660.84$ V $+ 500 \times 0.075 + j519.5 = 3467 - j1141.34$

$= 3650 \angle -18.2°$

$E_L = \sqrt{3} \times 3650 = 6321.98$ V

From the OCC, field current required at 6321.98 V is 71 A.

$I_r = 71 \angle 90° - 18.2° = 71 \angle 71.8°$

The current $I_{ar} = I_r$ is in phase with I_a. Hece, $I_{ar} = I_{ar} \angle 0° = 25 \angle 0°$

We have $I_f + I_{ar} = I_r$

∴ $I_f = 71 \angle 71.8° - 25 \angle 0° = 220.17 + 67.44 - 25 = -2.83 + j67.44 = 67.5 \angle 92.40°$

From the OCC, voltage (E_0) corresponding to this current (67.5) is 6000 V (line).

$$E_{0p} = \frac{6000}{\sqrt{3}} = 3464.10 \text{ V}$$

∴ $$\text{Voltage regulation} = \frac{E_{0p} - V_p}{V_p} = \frac{3464.10 - 3810.51}{3810.51} \times 100 = -9.09\%$$

(iii) 0.71 *power factor lagging:*

$\cos\phi = 0.71$, so $\phi = 44.76°$ and $I_a = 500 \angle 0°$

$V_p = V_p \angle + \phi = 3810.51 \angle 44.76 = 2705 + j2683.12$ V

$E = V_p + I_a Z_L = V_p + I_a (R_a + jX_L) = V_p + I_a R_a + jI_a X_L$

$= 2705 + j2683.12 + 500 \times 0.075 + j519.5 = 2705 + j2683.12 + 37.5 + j519.5$

$= 2742.5 + j3202.62 = 4216.40 \angle 49.42°$

$E_L = \sqrt{3} \times 4217 = 7304.05$ V

From the OCC, field current required at 6321.98 V is 95 A.

$I_r = 95 \angle 90° + 49.42 = 95 \angle 139.42$

The current I_{ar} is in phase with I_a. Hence, $I_{ar} = I_{ar} \angle 0° = 25 \angle 0°$

We have $I_f + I_{ar} = I_r$

∴ $I_f = 95 \angle 139.42 - 25 \angle 0° = -72.15 + j61.79 - 25 = -97.15 + j61.79 = 115 \angle 147.54$

From the OCC, voltage (E_0) corresponding to this current (115) is 7900 V (line).

∴ Voltage regulation $= \dfrac{7900 - 6600}{6600} \times 100 = 19.69\%$

Example .16 The following table gives the data for OCC and ZPFC for a 6-pole, 440 V, 50 Hz, 3-phase, star-connected alternator. The resistance between two terminals of the alternator is 0.3 Ω. Find the regulation of the alternator at full load current of 40 A at 0.8 power factor lagging using (i) synchronous impedance method, (ii) mmf method, (iii) ASA method, and (iv) ZPF method.

Field current	2	4	6	7	8	10	12	14	16	18
OCC volt	156	288	396	440	474	530	568	592	–	–
SC line current	11	12	34	40	46	57	69	80	–	–
ZPF terminal voltage	–	–	–	0	80	206	314	398	460	504

Solution: Terminal voltage per phase, $\dfrac{440}{\sqrt{3}} = 254$ V

Armature resistance per phase $= \dfrac{0.3}{2} = 0.15\ \Omega$

The OCC, SCC and ZPFC are shown in Fig. 49.

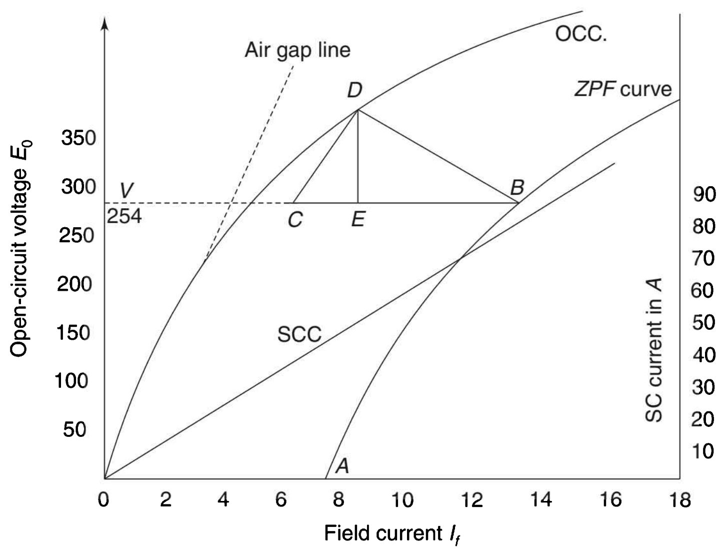


Fig. 49

(i) *Synchronous impedance method*

From the table, field current of 7 A gives the OC voltage of 440 V and the SC current is 40 A.

$$Z_s = \frac{\text{OC voltage}}{\text{SC current}} = \frac{254}{40} = 6.35\ \Omega$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{6.35^2 - 0.15^2} = 6.34\ \Omega$$

$$\cos\phi = 0.8, \text{ hence } \phi = -36.87^2,\ I_a = I_a \angle\phi° = 40 \angle -36.87°$$

$$E_p = V_p + I_a Z_s = 254\angle 0° + (40\angle -36.87°) \times (0.15 + j6.34)$$

$$= 254\angle 0° + (40\angle -36.87°) \times (6.34\angle 88.64°) = 254 + 253.6\angle 51.77°\text{ V}$$

$$= 254 + 156.93 + j199.21 = 410.93 + j199.21 = 456.67\angle 25.8°\text{ V}$$

$$\therefore \quad \text{Voltage regulation} = \frac{E_p - V_p}{V_p} = \frac{456.67 - 254}{254} \times 100 = 79.7\%$$

(ii) *MMF method*

From Fig. 50(a), field current required for OC voltage is 7 A, i.e., $I_{f1} = 7$

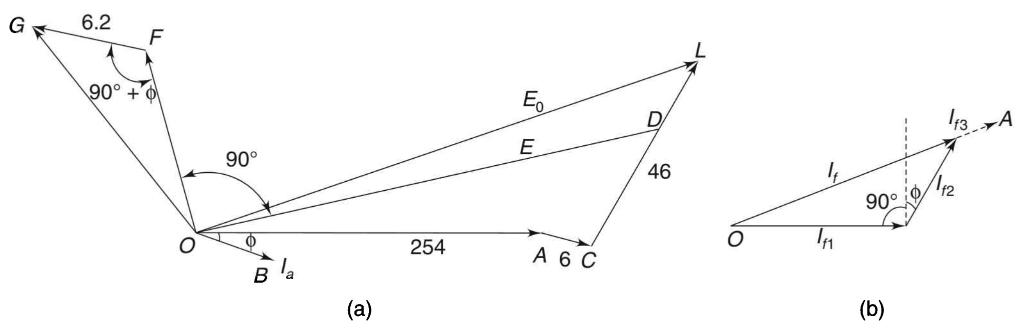


Fig. 50 Vector diagram for lagging power factor

Field current required to circulate full load current is 7 A, $I_{f2} = 7$

OA = 7 A, AB = 7 A and is at angle of (90° + ϕ)

$$\therefore \quad OB = \sqrt{7^2 + 7^2 + 2 \times 7 \times 7 \times \cos(\pi - (90° + \phi))}$$

$$= \sqrt{49 + 49 + 98\cos(90° - \phi)} = \sqrt{98 + 98\sin\phi} = \sqrt{98 + 98 \times 0.6} = 12.52\text{ A}$$

From the OCC, the OC phase voltage corresponding to a field current of 12.52 A is 330 V.

$$\text{Voltage regulation} = \frac{330 - 254}{254} \times 100 = 29.92\%$$

(iii) *ASA method*

According to Fig. 50(b), field current I_{f3} gives the value saturation effect.

So, total field current = $I_f + I_{f3}$

So, we know the value of I_f= 12.52 A and value of the I_{f3} is measured from the OCC and found 2.5 A

So, total field current required is 12.52 + 2.5 A

= 15.02 A. The voltage corresponding to this current can be obtained from the OCC and it is 350 V

$$\text{Voltage regulation} = \frac{350 - 254}{254} \times 100 = 34.90\%$$

(iv) *ZPF method:*

Draw a horizontal line at the rated phase voltage of 254 V which meets the ZPFC at point *B*. On this line, point *C* such that OA = BC = 7 A which is the field current required to circulate full load current on SC. From point *C*, draw a line parallel to air gap line which meets the OCC at point *D*. Draw a line DE perpendicular to the line BC. EB is the field current (I_{ar}) 6.2 A required to balance the armature reaction on load. ED gives the leakage impedance drop.

From Fig. 9.49, DE = 46 V, EB = 6.2 A.

$$\therefore \quad I_a X_L = 46, \quad X_L = \frac{46}{40} = 1.15\ \Omega$$

From Fig. 9.50(a), OA = 254 V, AC = $I_a R_a$ = 40 × 0.15 = 6 V, CD = 46 V

$$\therefore \quad OD = E = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_L)^2}$$

$$= \sqrt{(254 \times 0.8 + 6)^2 + (254 \times 0.6 + 46)^2} = \sqrt{209.2^2 + 198.4^2} = 288.3\ \text{V}$$

From OCC, the field current corresponding to 288.3 V is 9 A. This current leads *E* by 90 and is shown by OF. Now from the Fig. 54,

$$OG = \sqrt{9^2 + 6.2^2 + 2 \times 9 \times 6.2 \cos(90° + \phi)}$$

$$= \sqrt{81 + 38.44 + 111.6 \cos(\pi - (90° + \phi))}$$

$$= \sqrt{119.44 + 111.6 \sin\phi} = \sqrt{119.44 + 111.6 \times 0.6} = 13.65\ \text{A}$$

From OCC, the voltage corresponding to 13.65 A is 345 V.

$$\text{Regulation} = \frac{345 - 254}{254} \times 100 = 35.82\%$$

Example .17 A 3-phase star-connected 1800 kVA, 13500 V, alternator has armature synchronous resistance and reactance of 1.5 Ω and 30 Ω per phase, respectively. Find the % regulation for a load of 1300 kW at 0.8 leading power factor.

Solution: Load, $1300 \times 1000 = \sqrt{3} V_L I_L \cos\phi$

$$\therefore \quad I_L = \frac{1300 \times 1000}{\sqrt{3} \times 13500 \times 0.8} = 69.49\ \text{A}$$

$$\therefore \quad I_a R_a = 69.49 \times 1.5 = 104.23\ \text{V} \quad \text{and} \quad I_a X_s = 69.49 \times 30 = 2084\ \text{V}$$

Phase voltage, $V_p = \dfrac{13500}{\sqrt{3}} = 7794.22$ V

The phasor diagram is shown in Fig. 51.

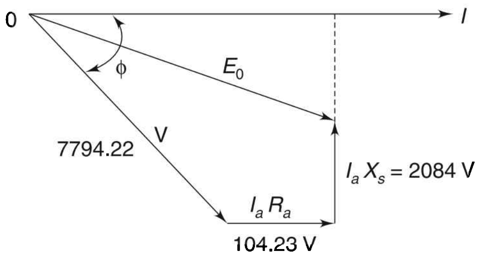


Fig. 51

From Fig. 51, $E_0 = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi - I_aX_s)^2}$

$$= \sqrt{(7794.22 \times 0.8 + 104.23)^2 + (7794.22 \times 0.6 - 2084)^2}$$

$$= \sqrt{6339.60^2 + 2592.53^2} = 6849.21 \text{ V}$$

$$\text{Regulation} = \frac{E_0 - V_p}{V_p} \times 100 = \frac{6849.21 - 7794.22}{7794.22} \times 100 = -12.12\%$$

Example .18 A 2500 kVA, 22 kV, 3-phase star-connected alternator has armature resistance of 0.3 Ω per phase and reactance of 6 Ω per phase. It delivers full load current at 0.8 power factor lagging at rated voltage. Find the terminal voltage for the same excitation and the load current at 0.8 power factor leading.

Solution: Full load current at 0.8 pf, $I_a = \dfrac{2500 \times 1000}{\sqrt{3} \times 22000 \times 0.8} = 82$ A

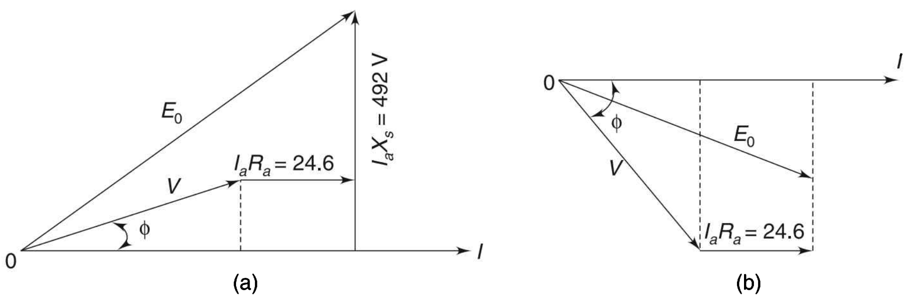


Fig. 52

Phase voltage, $V_p = \dfrac{22000}{\sqrt{3}} = 12701.70$ V

$$I_aR_a = 82 \times 0.3 = 24.6 \text{ V} \quad \text{and} \quad I_aX_s = 82 \times 6 = 492 \text{ V}$$

As seen from Fig. 9.52(a),

$$E_0 = \sqrt{(12701.70 \times 0.8 + 24.6)^2 + (12701.70 \times 0.6 + 492)^2} = 13021.13 \text{ V}$$

Now we know the E_0 and we are required to find the terminal voltage at the 0.8 power factor leading. Hence, from Fig. 52(b),

$$E_0^2 = (0.8\text{ V} + 24.6)^2 + (0.6\text{ V} - 492)^2 = 13021.13^2$$

$$V = 12529.08\text{ V}$$

Example .19 A 3-phase, 50 Hz, star-connected 2500 kVA, 2200 V alternator gives an SC current of 600 A for a certain excitation. With the same excitation, OC voltage is 900 V. The resistance between two terminals is 0.14 Ω. Find full load regulation at unity power factor and 0.8 lagging.

Solution:

$$Z_s = \frac{\text{OC voltage per phase}}{\text{SC current per phase}} = \frac{900/\sqrt{3}}{600} = 0.866\ \Omega$$

Rated voltage per phase, $$V_p = \frac{2200}{\sqrt{3}} = 1270\text{ V}$$

Per phase resistance, $$R_a = \frac{0.14}{2} = 0.07\ \Omega$$

$$\therefore \quad X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{0.866^2 - 0.07^2} = 0.86\ \Omega$$

$$\text{Full load current} = \frac{2500 \times 1000}{\sqrt{3} \times 2200} = 656.07\text{ A}$$

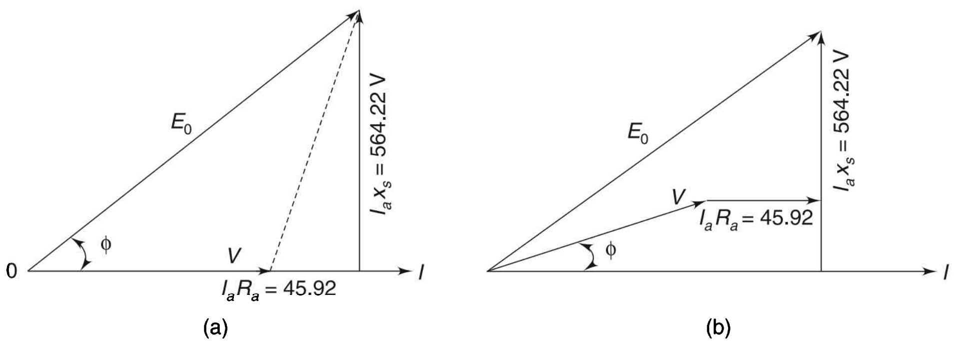


Fig. 53

$$I_aR_a = 656.07 \times 0.07 = 45.92\text{ V};\ I_aX_s = 564.22\text{ V}$$

(i) *Unity power factor:* See Fig. 53(a)

$$E_0 = \sqrt{(V\cos\phi + I_aR_a)^2 + I_aX_s^2} = \sqrt{(1270 + 45.92)^2 + 564.22^2} = 1431.77\text{ V}$$

$$\text{Regulation} = \frac{1431.77 - 1270}{1270} = 12.73\%$$

(ii) *0.8 power factor lagging:* See Fig. 53(b)

$$E_0 = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi + I_aX_s)^2}$$
$$= \sqrt{(1270 \times 0.8 + 45.92)^2 + (1270 \times 0.6 + 564.22)^2}$$
$$= \sqrt{1061.92^2 + 1326.22^2} = 1698.98 \text{ V}$$

$$\text{Regulation} = \frac{1698.98 - 1270}{1270} \times 100 = 33.77\%$$

Example .20 Effective resistance of a 1200 kVA, 3300 V, 50 Hz, 3-phase, star-connected alternator is 0.3 Ω per phase. A field current of 35 A produces a current of 200 A on SC and 1100 V (line) on open circuit. Find the power angle and pu change in terminal voltage when the full load of 1200 kW at 0.8 power factor lagging is thrown off. Also draw the phasor diagram.

Solution:
$$Z_s = \frac{\text{OC voltage per phase}}{\text{SC current per phase}} = \frac{1100/\sqrt{3}}{200} = 3.175\ \Omega$$

∴
$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{3.175^2 - 0.3^2} = 3.160\ \Omega$$

Rated phase voltage, $V_p = \dfrac{3300}{\sqrt{3}} = 1905$ V

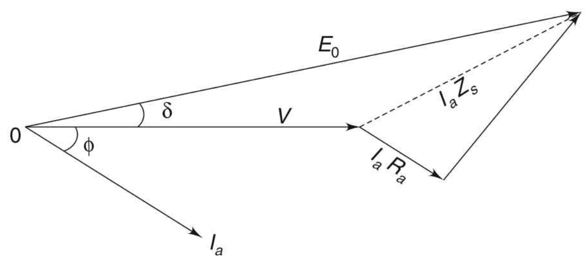


Fig. 54

$$\tan\delta = \frac{X_s}{R_a} = \frac{3.160}{0.3} = 10.53;\ \phi = 84.57°$$

Rated full load current, $I_a = \dfrac{1200 \times 1000}{\sqrt{3} \times 3300 \times 0.8} = 262$ A

Taking V_p as reference phasor, $V_p \angle 0° = 1905 \angle 0°$ and $I_a \angle -36.86° = 262 \angle -36.86°$

As per Fig. 54, $E_0 = V + I_aZ_s = 1905 \angle 0° + (262 \angle -36.86°) \times 3.175 \angle 84.57°$

$$E_0 = 1905 \angle 0° + 831 \angle 47.71° = 1905 + j0 + 559 + j614.73$$
$$= 2464 + j614.73 = 2539.52 \angle 14$$

∴ pu change in terminal voltage $= \dfrac{2539.52 - 1905}{1905} = 0.33$ pu

21.3 Types of Voltage Regulators

We know that when the alternator is loaded, its terminal voltage reduced due to armature reaction. So, its terminal voltage has to be kept constant equal to rated voltage from no load to full load condition. There is only one method by which it can be maintained constant and that is by controlling the field current of the alternator. There are two main types of voltage regulator used for this purpose.

(i) Electrical voltage regulator:

(a) Tirrill voltage regulator.

(b) Brown Boveri voltage regulator.

(ii) Electronics voltage regulator.

Tirrill voltage regulator controls the voltage very fast. In this regulator, the field regulating resistance is inserted or removed in steps. But the main problem is that it gives large oscillations in the armature circuit because of insertion or removal of resistance in the circuit. On the other hand, Brown Boveri regulator gives smooth operation because in that regulator, the field regulating resistance is inserted or removed very gradually and in small steps.

However, today electronics voltage regulator or say automatic voltage regulator are widely used for smooth and precise control of the voltage regulation.

22 POWER EQUATION OF A SYNCHRONOUS GENERATOR

To derive the various power equations, suppose that a cylindrical rotor type alternator is connected to the infinite bus as shown in Fig. 55(a). That means it is connected with constant voltage and constant frequency source.

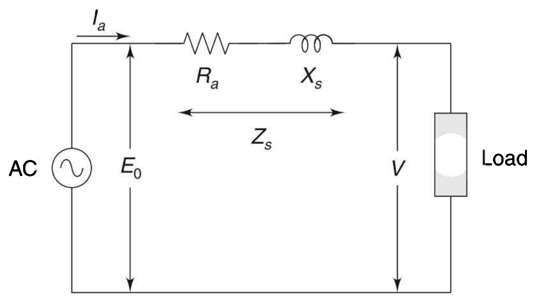


Fig. 55(a)

Let V = Terminal voltage per phase.

E_0 = Generated emf per phase.

I_a = Armature current.

δ = Phase angle between E_0 and V.

$\cos \phi$ = Lagging power factor of the alternator.

β = Impedance angle.

Taking V as reference, the phasor diagram for the lagging power factor is shown in Fig. 55(b). In alternator, the angle between induced emf E_0 and V is δ.

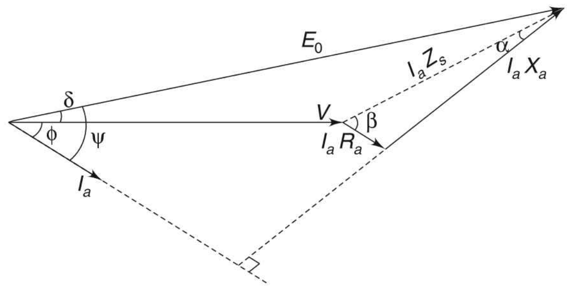


Fig. 55(b)

∴ $$V = V \angle 0 \quad \text{and} \quad E_0 = E_0 \angle \delta$$

The synchronous impedance is given by

$$Z_s = R_a + jX_s$$

Here, $$\beta = \tan^{-1}\frac{X_s}{R_a} \quad \text{and} \quad \alpha = 90° - \beta = \tan^{-1}\frac{R_a}{X_s}.$$

By applying KVL in Fig. 59(a),

$$E_0 = V + Z_s I_a$$

∴ $$I_a = \frac{E_0 - V}{Z_s}$$

Complex output power of the generator per phase

$$S_{GO} = P_{GO} + Q_{GO} = VI_a$$

$$= V\left(\frac{E_0 - V}{Z_s}\right)$$

$$= V\angle 0°\left(\frac{E_o \angle\delta - V\angle 0°}{Z_s \angle\beta}\right) \text{(Putting the values of } E_0, V \text{ and } Z_s\text{)}$$

$$= V\angle 0°\left(\frac{E_0}{Z_s}\angle\delta - \beta - \frac{V}{Z_s}\angle -\beta\right)$$

$$= \frac{VE_o}{Z_s}\angle\beta - \delta - \frac{V^2}{Z_s}\angle -\beta$$

$$P_{GO} + Q_{GO} = \frac{VE_o}{Z_s}\cos(\beta - \delta) + j\frac{VE_o}{Z_s}\sin(\beta - \delta) - \frac{V^2}{Z_s}(\cos\beta + j\sin\beta) \quad (.22.1)$$

From the above equation, real part gives the real output of the power per phase

$$P_{GO} = \frac{VE_o}{Z_s}\cos(\beta - \delta) - \frac{V^2}{Z_s}\cos\beta$$

Since $\cos\beta = \frac{R_a}{Z_s}$, putting these values, we get

$$P_{GO} = \frac{VE_o}{Z_s}\sin(\delta + \alpha) - \frac{V^2}{Z_s^2}R_a \quad (22.2)$$

Reactive output power per phase of the alternator

If we equating imaginary part to Eqn. (22.1), we get reactive power

$$Q_{GO} = \frac{VE_0}{Z_s}\sin(\beta - \delta) - \frac{V^2}{Z_s}\sin\beta$$

Since $\sin\beta = \frac{X_s}{Z_s}$ and $\beta = (90° - \alpha)$, putting these values, we get

$$Q_{GO} = \frac{VE_o}{Z_s}\cos(\delta + \alpha) - \frac{V^2}{Z_s^2}X_s \quad (22.3)$$

Similarly, complex power input to the generator per phase is found and from that real power input and reactive power input to the generator is found as under.

Complex input power to the generator per phase

$$S_{GI} = \frac{E_0^2}{Z_s}\cos\beta + j\frac{E_0^2}{Z_s}\sin\beta - \left(\frac{VE_0}{Z_s}\cos(\beta + \delta) + j\frac{VE_0}{Z_s}\sin(\beta + \delta)\right) \quad (22.4)$$

Real input power to the generator per phase

$$P_{GI} = \frac{E_0^2}{Z_s}R_a + \frac{VE_0}{Z_s}\sin(\delta - \alpha) \quad (22.5)$$

Reactive input power to the generator per phase

$$Q_{GI} = \frac{E_0^2}{Z_s^2}X_s - \frac{VE_0}{Z_s}\cos(\delta - \alpha) \quad (22.6)$$

The alternator is driven by the prime mover. So, the mechanical power input to alternator is the sum of the power input to the alternator and rotational loss. The rotational loss includes the friction and windage loss and core loss. We know that a synchronous generator converts mechanical energy into electrical energy. The prime mover used may be a diesel engine, steam turbine or water turbine. Whatever the prime mover is used, basically its speed must remained constant regardless of the load. If it does not so, the power system frequency will change.

We also know that all the mechanical power given to generator is not converted into electric power because of losses of the machine. A power flow diagram is shown in Fig. .56.

The input mechanical power is the shaft power in the generator which is converted into electrical power which is given by

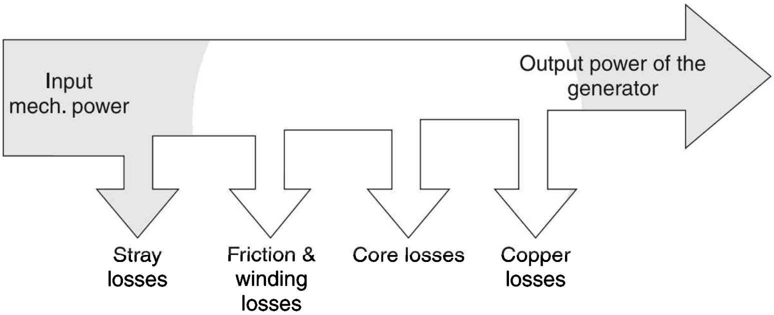


Fig. .56 Power flow diagram for alternator

$P_{con} = P_{GI} = \tau_{ind}\,\omega_m = 3E_0\, I_a \cos\psi$, where ψ is the angle between E_0 and I_a.

The real electric output power of the generator in line quantity given by

$$P_{GO} = \sqrt{3}V_L\, I_L \cos\phi = 3\ VI_a \cos\phi$$

Similarly, the reactive power can be given as in line and phase quantity as

$$Q_{GO} = \sqrt{3}V_L\, I_L \sin\phi = 3\ VI_a \sin\phi$$

If the armature resistance is neglected (since $X_s >> R_a$) then a very useful equation can be derived to approximate the output power of the generator. For this, refer to the phasor diagram shown in Fig. 57.

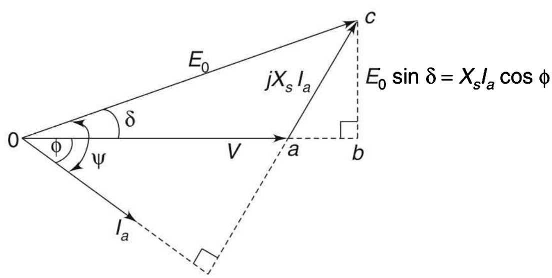


Fig. 57

Notice that the vertical part *bc* can be expressed as either $E_0 \sin\delta$ or $X_s I_a \cos\phi$,

Hence,
$$I_a \cos\phi = \frac{E_0 \sin\delta}{X_s}$$

$$\therefore\ P_{GO} = \frac{3VE_0 \sin\delta}{X_s} \quad \text{and} \quad \frac{VE_0 \sin\delta}{X_s} \text{ is the per phase power} \qquad (.22.7)$$

The above equation shows that the power developed by a generator depends on the angle δ between V and E_0. This angle is called torque angle of the machine.

In the same way, we can get the following equations:

$$Q_{GO} = \frac{VE_0}{X_s}\cos\delta - \frac{V^2}{X_s} \tag{22.8}$$

$$P_{GI} = \frac{VE_0}{X_s}\sin\delta = P_{GO} \tag{22.9}$$

$$Q_{GI} = \frac{E_0^2}{X_s} - \frac{VE_0}{X_s}\cos\delta \tag{22.10}$$

$$P_{GO(max)} = P_{GI(max)} \tag{22.11}$$

The maximum power indicated by the above equation is called static stability limit of the generator. Normally, real generators never even come close to this limit. Full load torque angle of 15 to 20 degrees are more typical of real machines.

23 MAXIMUM POWER OUTPUT OF THE SYNCHRONOUS GENERATOR

To obtain the maximum power output of the generator,

$$\frac{dP_{GO}}{d_\delta} = 0 \quad \text{and} \quad \frac{d^2P_{GO}}{d\delta^2} < 0$$

$$\frac{d}{d\delta}\left(\frac{VE_o}{Z_s}\sin(\delta+\alpha) - \frac{V^2}{Z_s^2}R_a\right) = 0$$

Since V, E_0, Z_s, R_a are constant,

$$\cos(\delta+\alpha) = 0 \text{ or } (\delta+\alpha) = 90°$$

$$\therefore \qquad \delta = 90° - \alpha$$

$\therefore$ $P_{GO} = \dfrac{VE_0}{Z_s} - \dfrac{V^2}{Z_s^2}R_a$. This occurs at $\delta = \theta$

Power output is maximum when
Load angle δ = Impedance angle β (23.1)

24 TORQUE DEVELOPED BY THE ALTERNATOR

The induced torque in the generator can be given as

$$T_{ind} = kB_{ROTOR} \times B_{STATOR}$$

or $= kB_{ROTOR} \times B_{net}$ and the magnitude of the torque is given as

$$T_{ind} = k\,B_{ROTOR} \times B_{net}\sin\delta \tag{24.1}$$

where B_{ROTOR} is the rotor flux and B_{net} is the resulting flux in the air gap of the machine. B_{ROTOR} produces voltage E_0, and B_{net} produced voltage V. The angle δ between E_0 and V is called power angle.

The torque induced in the generator can be expressed in electrical quantity as

$$T_{ind} = \frac{3VE_0 \sin \delta}{\omega_m X_s} \qquad (. \ 24.2)$$

Equation (9.24.1) gives the torque induced in the machine in terms of magnetic quantities while Eqn. (9.24.2) gives the same information in terms of electrical quantity.

.25 POWER ANGLE CHARACTERISTICS

We know that power output per phase of cylindrical rotor synchronous alternator is given as:

$$P_{GO} = \frac{VE_0}{X_s} \sin \delta$$

The power angle characteristic of the round rotor is shown in the Fig. ` 58. The alternator delivers maximum power when δ = 90°. If this angle becomes greater than 90°, the machine will lose synchronism. This generates large current and mechanical force. The dotted portion is the unstable portion.

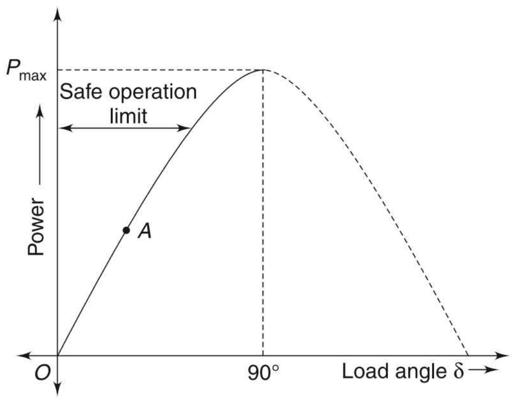


Fig. ..58 Power angle curve of alternator

This angle corresponds between field flux and the stator generated rotating flux. The maximum power of the alternator decides the static stability limit of the system. Safe operation always requires 15–20% power reserve.

Suppose, an alternator is running at a point A on the curve. If due to sudden spike of mechanical input, the load angle δ increases by a small value, it produces a torque which is not balanced by driving torque once the spike passes. The alternator returns to the point A due to this torque which slows down the rotor. This torque responsible for steady state operation is known as *synchronizing torque* and the power is called as synchronizing power.