1 INTRODUCTION

The first poly-phase induction motor was invented by Nikola Tesla in 1886. After this invention, the whole scenario of the engineering changed. It is believed that the most widely used motor is AC motor. More than 90% of the mechanical power used in industry is supplied by the poly-phase induction motor. The USA having a total 150 million horse power, uses more than 50 million horse power by induction motors alone. Each year, one million motors are added there. Moreover, 20 million single-phase fractional horse power motors are used in domestic application such as fans and washing machines.

The poly-phase induction motor does not require excitation except the AC source. You can say it is a singly excited machine. It requires reactive power and draws a lagging current. The power factor of the motor at the rated load is nearly 0.8 which is quite low at light loads. To mit the reactive power and keep the air gap short, the magnetic reactance of the induction otor must be high as compared to that of the synchronous motor of the same size and rating, except in the small motor category. This short air gap is determined by some factors like noise and magnetic losses.

Such motors are also known as asynchronous machines because they run at other than the synchronous speed. Like other electrical machines, an asynchronous machine is also reversible, i.e., it can operate as both a motor and a generator. The operation of the asynchronous machine is determined by the speed of the rotating field related to the rotor.

Just like other motors, it converts electrical power into mechanical power. In DC motors, the electrical power is directly conducted to the armature via brush and commutator, hence they are called conduction motors. In AC motors, the rotating part called the rotor does not get power directly as in DC motors. It comes through induction as in transformers. Hence, this motor is known as induction motor. However, the induction motor is a rotating transformer in which primary winding is stationary and secondary winding is rotating.

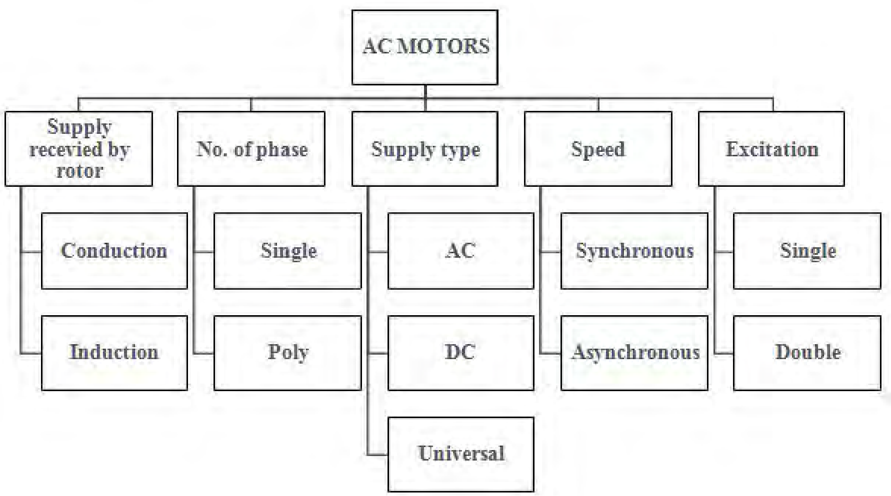
1.1 Classification of AC Motors

AC motors can be classified as in the flow chart given below.

1.2 Advantages of the Induction Motor

(i) Simple and robust construction.

(ii) Cheap and reliable.



(iii) Efficiency is higher because no brush and slip rings are needed, so friction losses are reduced.
(iv) No extra prime mover is required means it is self starting.
(v) Maintenance is low.

1.3 Disadvantages of the Induction Motor

(i) Its speed decreases as load is increased.
(ii) Starting torque is less as compared to the DC motors.

2 CONSTRUCTION

The construction of the poly phase induction motor is very simple as shown in Fig. ' 1. Its main parts are:

(i) A frame to support the stator and to carry bearings.
(ii) A laminated stator core to carry the poly phase winding.
(iii) A laminated rotor core to carry either squirrel cage or poly phase winding.
(iv) Shaft to maintain air gap.
(v) A cooling arrangement.

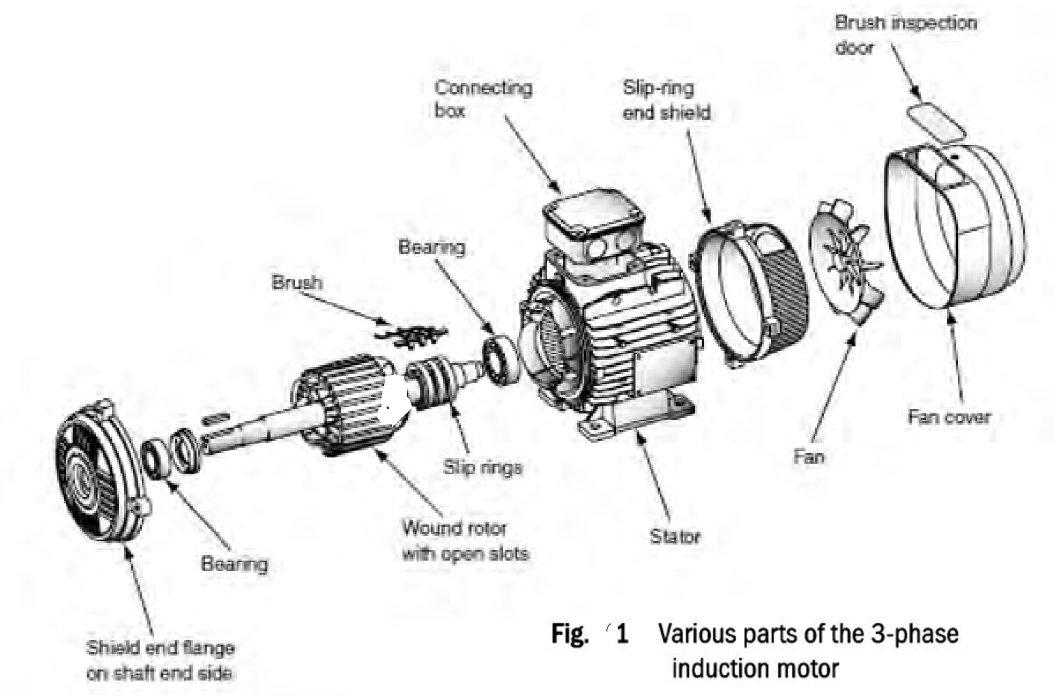


Fig. ′1 Various parts of the 3-phase induction motor

.2.1 Frame

It is the outer body of the motor as shown in Fig. .2. Its main function is to support the stator core and windings, to protect the inner parts and provide the facility for cooling. The frame may be die-cast or fabricated. Up to 50 kW rating, the frames are die-cast because this facilitates the use of thicker cross-section where more mechanical strength is required. For large motors like 250 kW and more, the frame is generally fabricated. The frame is provided with feet, and it must be enough strong and rigid because the air gap in the induction motor is very small. If the frame is not rigid the rotor will not remain concentric with the stator and will unbalance the magnetic pull.

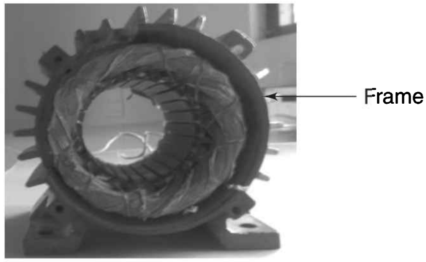


Fig. .2 Frame of poly phase induction motor

.2.2 Stator

The stator of the induction motor is just similar to that of the synchronous machine. The main function of the stator core is to carry the alternating flux which produces eddy current and hysteresis loss inside it. Hence, the core is made of high grade alloy steel lamination to reduce the eddy current losses. These laminations are slotted in the inner periphery of the frame and are insulated from one another by an oxide coating produced by heat treatment or by varnish coating. These laminations are held by the flange. Ventilating ducts are provided along the length of the core at every 5 or 7 cm by spacers placed between laminations. The insulated conductors are placed in the stator slots to form a three-phase winding. The winding may be either star- or delta-connected. Such stator is shown in Fig. .3.

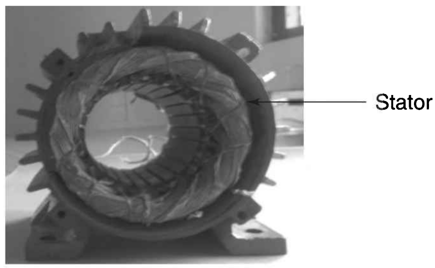


Fig. ˉ3 Stator of poly phase induction motor

Double layer windings are mostly used for the stator winding because they are very easy to manufacture, assemble and repair. The windings are distributed and are mostly short pitched. The short pitched and distributed windings are effective to limit the magnitudes of the harmonics in the air gap flux. Wherever possible, integral short winding is also used.

Now, air gap reluctance is different at different points of the stator while the rotor rotates. This pulsating reluctance produces pulsating exciting current, irregular torque, noise, etc. So the general tendency is to use a large number of stator slots to reduce the effect of variable air gap but if we increase the number of slots, the narrow teeth and manufacturing cost increases. So, the number of slots on the rotor is made different from that of stator slots and rotor slots are skewed to make more uniform reluctance.

If we conclude the above things, the windings, air gap and slots must be selected so that the exciting current and machine reactance gives the desired operation. The air gap length should be made as small as possible to reduce magnetizing current required to set up the air gap flux. If the air gap becomes too small, it increases the noise and tooth face loss and even poses a starting problem.

2.3 Rotor

The rotor of the induction motor is also built of laminations of the same material as the stator but the laminations are thicker than in stator because of the lower frequency of the rotor flux. The laminated cylindrical core is directly mounted on the shaft. These laminations are slotted on its outer periphery for rotor conductors. There are two main types of the rotor.

2.3.1 Cage rotor

This type of rotor consists of cylindrical laminated core having slots nearly parallel to the shaft axis or skewed. A copper or aluminium bar conductor which is insulated is placed in each slot. These bars are short circuited by heavy end rings of the same material. So the conductors and the end rings make a cage. Hence, this type of the rotor is called squirrel cage rotor as shown in Fig. 4.

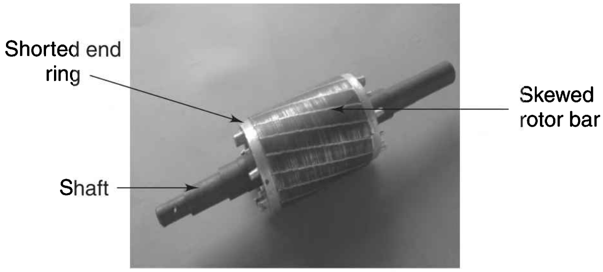


Fig. .4 Squirrel cage rotor

Skewing of the rotor conductors has the following advantages.

(a) More uniform torque is produced and the noise is reduced.
(b) Locking of the rotor is reduced.

Advantages of cage rotor

(i) Simple in construction.
(ii) No chance of burning of rotor winding.
(iii) Automatically equals the number of poles as the number of stator poles.
(iv) No maintenance, hence long life.
(v) Highly efficient.
(vi) Excellent running performance.

Disadvantages of cage rotor

(i) Produces low starting torque.
(ii) No speed control can be obtained from rotor side as rotor conductors are permanently shorted.

(iii) Draws high starting current.
(iv) Low power factor operation.

2.3.2 *Wound rotor (slip ring rotor)*

This type of rotor consists of an armature having slots on its outer periphery. Insulated conductors are placed in these slots and connected to form a three-phase distributed winding just similar to that of the stator winding. The rotor windings are connected in a star. The number of poles on the stator and on the rotor must be the same for producing torque.

The three terminals of star connections are connected to the three insulated slip rings as shown in Fig. 5. These slip rings are mounted on the rotor shaft having brushes resting on them. The brus.ıes are connected to the three variable résistances which are connected in star form. These resistances have the following purpose.

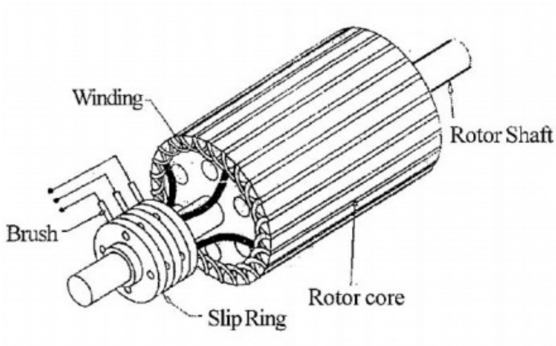


Fig. 5 Wound rotor (slip ring rotor)

(a) To increase the starting torque of the motor with reduced starting current.
(b) To control the speed of the motor.

Advantages of wound rotor

(i) Gives high starting torque.
(ii) External resistance can be inserted in rotor winding.
(iii) Draws low starting current.
(iv) Can be started from rotor side.
(v) Speed control is possible both from the rotor and stator side.
(vi) Better power factor operation compared to the cage motor.

Disadvantages of wound rotor

(i) Construction is not simple.
(ii) Rotor winding can be damaged.
(iii) Require more maintenance compared to cage motor.
(iv) More costly.
(v) Less efficient than cage motor.
(vi) Running performance is inferior to cage motor.

A comparison between cage rotor and wound rotor is tabulated here.

Sl No.	Cage rotor	Wound rotor
01	Robust and cheaper	Costlier than cage rotor
02	The absence of the brush and slip rings gives no sparking	The brush and slip rings gives sparking problems
03	Lesser maintenance	Higher maintenance than cage rotor
04	Higher starting current and lower starting torque	Higher starting torque and lower starting current
05	Additional resistance can't be inserted in the rotor circuit	Additional resistance can be inserted in the rotor circuit

3 PRODUCTION OF ROTATING MAGNETIC FIELD

When a balanced 3-phase current is flowing in the 3-phase winding, a rotating magnetic field is produced. All 3-phase rotating machines are associated with the rotating magnetic field in their air gaps.

A stator of a 2-pole, 3-phase machine has three windings displaced from each other by 120° along the air gap periphery. Each phase is distributed or spreads over 60°, i.e., phase spread, $\sigma = 60°$ as in Fig. 6(a). The three-phase winding *a*, *b* and *c* are represented by three full pitched coils *aa′*, *bb′* and *cc′*.

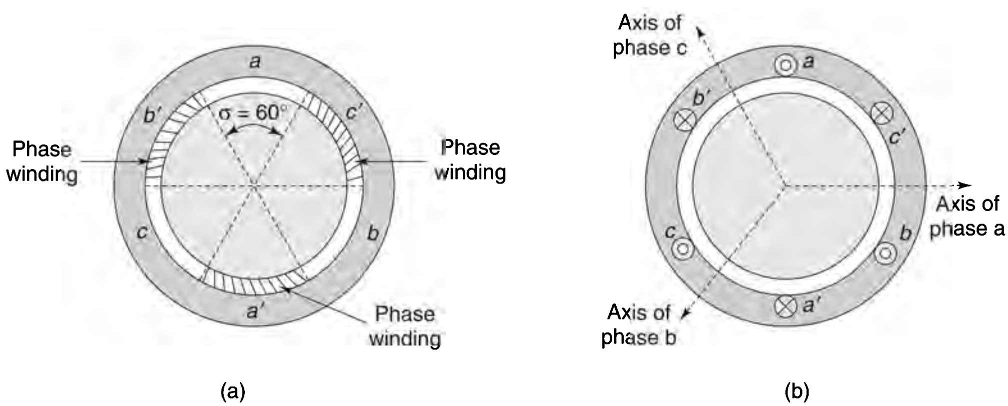


Fig. 6

A current in phase winding sets up magnetic flux directed along the magnetic axis of the coil *aa′*. Here, it is assumed that positive currents are flowing, indicated by crosses in the coil sides of *a′*, *b′*, *c′* in Fig. 6(b). The current produced by three-phase winding is shown in Fig. 6(c).

- Now, at the instant 1, current in the phase *a* is positive and maximum, i.e., I_m and $i_b = i_c = -\frac{I_m}{2}$

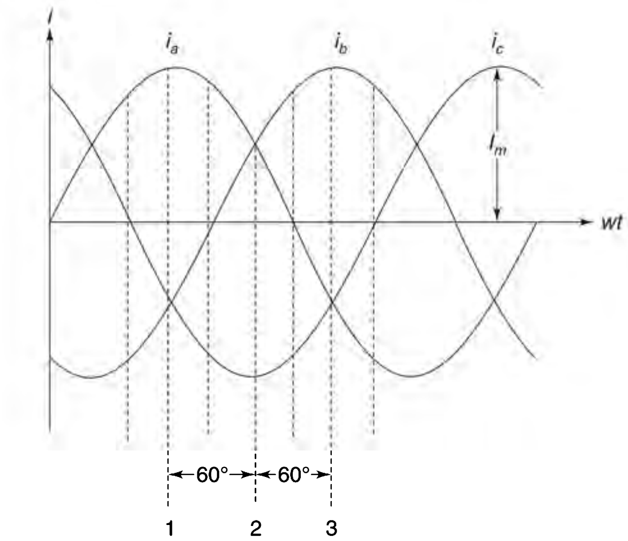


Fig. ' 6(c)

- At the instant 2, $i_a = \frac{I_m}{2}, i_b = \frac{I_m}{2}$ and $i_c = -I_m$.
- At the instant 3, $i_a = -\frac{I_m}{2}, i_b = I_m$ and $i_c = -\frac{I_m}{2}$.

The two poles produced by the resultant flux are shown in Fig. 7(a) which is turned through a further 60°. The space angle traversed by the resultant flux is equal to the time angle traversed by the currents.

The rotating field speed, for P pole machine is $\frac{1}{P/2}$ revolution in one cycle.

- Hence, $\frac{f}{p/2}$ revolution in f cycle.
- $\frac{f}{p/2}$ revolution in one second. (because f cycles are completed in one second)
- So, synchronous speed, $N_s = \frac{120f}{p}$ rpm

This can also be achieved by graphical method or say by space phasor representation as in Fig. '.7(b).

- When currents i_a, i_b, and i_c flow in their respective field windings, then three stationary pulsation mmf F_a, F_b, and F_c produced which gives resultant mmf F_R which is rotating at synchronous speed.
- At the instant 1,

$$i_a = I_m \rightarrow \text{Space phasor } F_a = \text{maximum mmf } F_m$$

$$i_b = i_c = -\frac{I_m}{2} \rightarrow \text{the mmf phasor } F_b = F_c = \frac{F_m}{2}$$

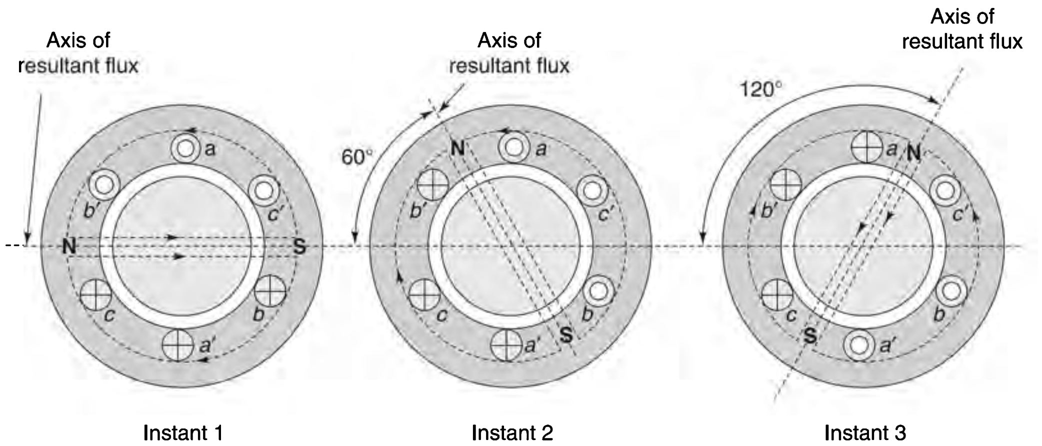


Fig. 7(a)

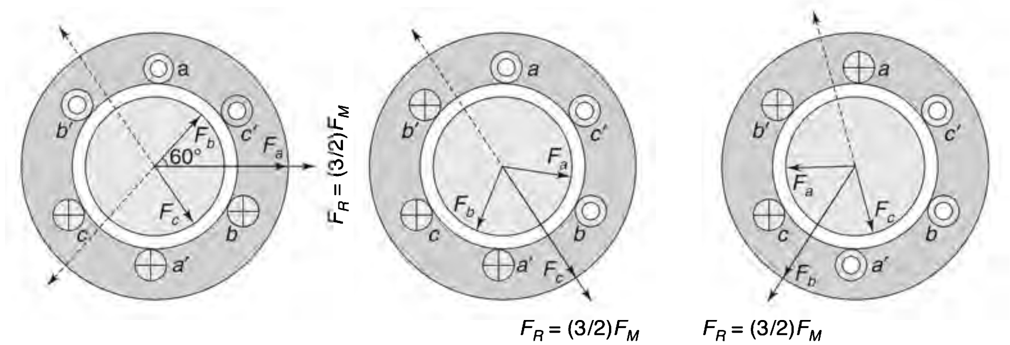


Fig. 7 (b) Graphical representation

The resultant of F_a, F_b, and F_c is F_R and is magnitude is given by

$$F_R = F_m + \frac{2F_m}{2}\cos 60° = \frac{3}{2}F_m$$

The vertical component of F_b and F_c cancels each other.

- At the instant 2

$$i_a = i_b = \frac{I_m}{2} \quad \text{and} \quad i_c = -I_m$$

The mmf phasor $F_a = F_b = \dfrac{F_m}{2}$ and space phasor F_c = max. mmf F_m

The resultant mmf $F_R = \dfrac{3}{2}F_m$ (it rotates by a space angle of 60° clockwise)

- At the instant 3,

$$i_a = i_c = -\frac{I_m}{2} \quad \text{and} \quad i_b = I_m$$

The resultant mmf $F_R = \frac{3}{2} F_m$ (it rotate further by a space angle of 60° clockwise from its position at instant 2).

So, from the above discussion, we can conclude the following points:

Some Facts on Rotating Magnetic Field and Flux

(i) When the three-phase AC supply is given to the stator of three-phase induction motor, rotating magnetic flux is produced in the air gap.

(ii) The magnitude of the rotating flux is constant and equal to the $\frac{3}{2} \phi_m$.

(iii) The direction of the rotating flux is the same as the phase sequence of the supply.

4 OPERATING PRINCIPLE OF 3-PHASE INDUCTION MOTORS

As said earlier, the general principle of the induction motor is to convert electrical energy into mechanical energy. When we apply 3-phase AC supply to the stator of the induction motor, the rotating magnetic field of constant magnitude is produced in the air gap which rotates at the synchronous speed. Its rotating direction depends upon the phase sequence of the applied three-phase voltage. This rotating flux is cut by the rotor conductors which are stationary. Let for simplicity, one conductor be on the rotor as shown in Fig. 8(a). The rotation of the rotating field is clockwise. According to Faraday's law of electromagnetic induction, a voltage is induced in the rotor conductor. Since the rotor circuit is closed, the current will flow in the rotor conductor. Now, when the rotating field is rotating in the clockwise direction and the conductor is stationary, we can say that the conductor is moving in the anticlockwise direction with respect to the rotation field. By right hand rule we can find the direction of the current to be outward as shown by the dot as in Fig. 8(b).

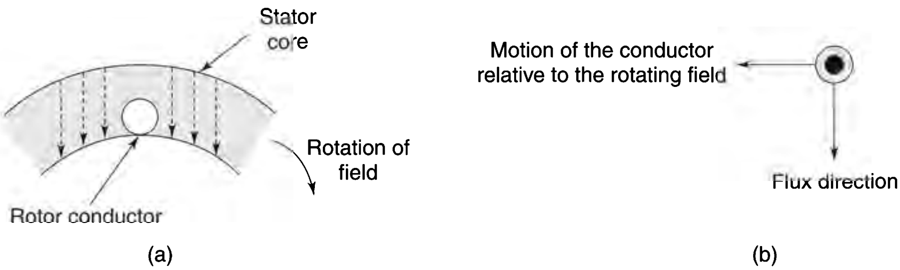


Fig. 8 Section of alternator

Now the rotor conductor current produces its own magnetic field as in Fig. 8(c). We already know that if a conductor which draws a current, is placed in a magne.ıc field, a force is experienced by it. So, the rotor conductor experiences a force whose direction can be found by the left hand rule as in Fig. 8(d). It is seen that the force of the conductor is in the same direction as that of the rotating field. The conductor being in the rotor slots on the outer periphery of the rotor, this force acts in a tangential direction and the rotor starts to rotate in the direction of the rotating magnetic field.

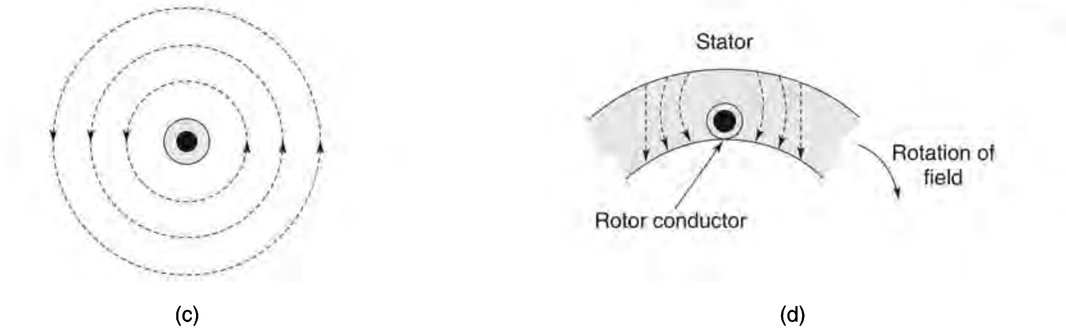


Fig. 8

The other way of understanding the production of torque in the induction motor is by the torque equation,

$$T_{\text{ind}} = -\frac{P}{2} K M_{\text{STATOR}} M_{\text{ROTOR}} \sin \delta$$

Here, T_{ind} is the induced torque, P is the number of the pole of the machine, K is constant, M_{STATOR} = stator mmf, M_{ROTOR} = rotor mmf and δ = angle between the two fields. The minus sign indicates that the fields tend to align themselves by decreasing the angle δ.

From the above equation it is clear that a constant torque which varies neither with time nor with rotor position can be obtained when two mmfs having constant amplitude and constant angular displacement from each other are possible. It is very easy to get the two mmfs of constant amplitude but very difficult to get constant angle between stator and rotor mmf axis, especially when one winding is stationary and the other is rotating. This can be achieved in three ways (i) both stator and rotor axes are made to fix in the space even when the rotor is rotating, (ii) if the rotor mmf axis is fixed relative to the rotor, the stator mmf axis must rotate at the rotor speed relative to the stationary stator winding, and (iii) the two mmf axes rotate such that they remain stationary with respect to each other.

In the induction motor, the stator mmf rotating at the synchronous speed N_s produces poly-phase current in the rotor winding. Since the rotor is stationary, the frequency of the rotor current is the same as that of the stator supply. Therefore, the rotor mmf also rotates at the synchronous speed N_s in the direction of the stator mmf.

> Hence, the stator magnetic field and the rotor magnetic field are stationary with respect to each other irrespective of the rotor speed.

So, interaction of the two relatively stationary fields produces torque as per the above equation. Due to this torque, rotor begins to rotate at the speed N in the direction of the rotating magnetic field to reduce the relative motion as per Lenz's law.

It is obvious that when $N = N_s$, no voltage, and hence no current, is produced in the rotor circuit, and hence no torque is produced.

5 SLIP

From above discussion, it is clear that the induction motor can never run at the synchronous speed. It runs at slightly less than the synchronous speed. Therefore, the induction motor is called asynchronous motor. The difference between the synchronous speed N_s and the actual rotor speed N is called the slip.

Hence, $$\text{Slip } s = \frac{\text{Synchronous speed} - \text{Actual speed of the rotor}}{\text{Synchronous speed}}$$

$$\text{Slip, } s = \frac{N_s - N}{N_s} = \frac{\omega_s - \omega_r}{\omega_s} \qquad (5.1)$$

Hence, $$\% \text{ slip } s = \frac{N_s - N}{N_s} \times 100$$

Sometimes, $N_s - N$ is called the slip speed. So, the actual speed of the rotor or motor is $N = N_s(1 - s)$.

In small motors, the value of slip at full load changes from 5% while in large motors, its value is 2%.

It must be kept in mind that rotating flux rotates at the synchronous speed relative to the stator but at slip speed relative to the rotor.

.6 FREQUENCY OF THE ROTOR CURRENT

When the rotor is stationary, the frequency of the rotor current is same as that of a supply frequency. But when rotor starts to rotate, the frequency of the rotor depends upon the slip speed. So if the frequency of the rotor at any slip speed is f_r, then

$$N_s - N = \frac{120 f_r}{p}. \text{ And also, } N_s = \frac{120 f}{P}$$

Dividing one equation by the other we get, $\frac{f_r}{f} = \frac{N_s - N}{N_s} = s$

$$\text{Rotor frequency, } f_r = sf \qquad (6.1)$$

When the rotor is stationary, $N = 0$, so Slip $s = 1$ and $f_r = f$

If the motor is driven by a prime mover up to the synchronous speed, N_s then $s = 0$ and $f_r = 0$. Therefore, the frequency of the rotor varies from $s = 0$ to $s = 1$.

Example .1 A 3-phase, 50 Hz, 4-pole induction motor has a full load speed of 1460 rpm. Find the slip percentage.

Solution:

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\text{Slip} = \frac{N_s - N}{N_s} = \frac{1500 - 1460}{1500} = 0.02 = 2\%$$

Example .2 The frequency of the emf in the stator of a 4-pole induction motor is 50 Hz and that of the rotor is 2 Hz. Find the slip and actual speed of the motor.

Solution: Rotor frequency, $f_r = sf$

$$\therefore \quad \text{slip } s = \frac{f_r}{f} = \frac{2}{50} \; 0.04 = 4\%$$

Now,
$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Speed of the motor $N = (1 - s)\, N_s = (1 - 0.04) \times 1500 = 1440$ rpm

Example .3 A 3-phase, 8-pole, 60 Hz, induction motor has a slip of 1% at no load and 3% at full load. Find (i) synchronous speed, (ii) no load speed, (iii) full load speed, (iv) frequency of the rotor current at standstill and (v) frequency of the rotor current at full load.

Solution:

(i) Synchronous speed $N_s = \frac{120f}{P} = \frac{120 \times 60}{8} = 900$ rpm

(ii) No load speed, $N_0 = (1 - s_0)\, N_s = (1 - 0.01) \times 900 = 891$ rpm

(iii) Full load speed, $N_{FL} = (1 - s_{FL})\, N_s = (1 - 0.03) \times 900 = 873$ rpm

(iv) Frequency of the rotor current at standstill,

$$f_r = sf = 1 \times 60 = 60$$

(v) Frequency of the rotor current at full load,

$$f_r = s_{FL} f = 0.03 \times 60 = 1.8 \text{ HZ}$$

Example .4 A 3-phase, 6-pole, induction motor is connected with 50 Hz supply. If the full load slip is 3%, find the (i) motor speed, (ii) frequency of the rotor current at full load.

Solution:

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

(i) Motor speed, $N = (1 - s)\, N_s = (1 - 0.03) \times 1000 = 970$ rpm

(ii) $f_r = sf = 50 \times 0.03 = 1.5$ Hz

7 ROTOR EMF AND ROTOR CURRENT

When the rotor is stationary, an induction motor is just like a transformer with secondary short circuited. Hence, induced emf per phase in the rotor E_2 at the starting is given by,

$$E_2 = E_1 \times \frac{N_2}{N_1},$$

where E_1 = induced emf per phase to stator
N_1 = number of turns of stators
N_2 = number of turns of rotor

When the rotor is running, its relative speed with respect to the stator field is less. Hence, the induced emf in the rotor which is directly proportional to the slip is also less, and is given by sE_2.

When the rotor conductors of the induction motor are cut by the rotating magnetic field, an emf is induced in the rotor conductor according to Faraday's law and hence rotor current flow. This rotor current can be calculated as under.

(i) When the rotor is at standstill position.

Let E_2 = *emf induced in the rotor conductor per phase at standstill*
R_2 = *rotor resistance per phase*
X_2 = *rotor reactance per phase at standstill* = $2\pi f L_2$
Z_2 = *rotor impedance per phase at standstill*
I_2 = *rotor current per phase at standstill*

So, $$Z_2 = R_2 + jX_2.$$

$$\text{Rotor current at standstill, } I_2 = \frac{E_2}{Z_2} \qquad (7.1)$$

$$\text{Power factor at standstill, } \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

(ii) Rotor current at slip s or, say, in running condition,

The emf induced in the rotor conductor per phase at slip, $s = E_{2s} = s\,E_2$

Rotor resistance per phase = R_2

Rotor reactance per phase at slip s, $X_{2s} = 2\pi s f_r L_2 = s\,X_2$

Rotor impedance per phase at slip s, $Z_{2s} = R_2 + jsX_2$

$$\therefore \quad \text{Rotor current per phase } I_{2s} = s\frac{E_2}{Z_{2s}} = \frac{E_2}{\sqrt{\left[\frac{R_2}{s}\right]^2 + X_2^2}} \qquad (7.2)$$

$$\text{Power factor at slip } s, \cos\phi_{2s} = \frac{R_2}{\sqrt{R_2^2 + (sX_2^2)}} = \frac{R_2}{\sqrt{\left[\frac{R_2}{s}\right]^2 + X_2^2}}$$

So, if we make a comparison of the various rotor parameters at different conditions, we can conclude the following:

Rotor parameters	***Motor at starting condition***	***Motor at running condition***
Per phase rotor induced emf	$E_2 = E_1 \times \frac{N_2}{N_1}$	$sE_2 = s\left(E_1 \times \frac{N_2}{N_1}\right)$
Frequency of rotor emf	f	$f_r = sf$
Rotor resistance per phase*	R_2	R_2
Rotor reactance per phase	$X_2 = 2\pi f L_2$	$sX_2 = 2\pi sf L_2$
Rotor impedance per phase	$Z_2 = R_2 + jX_2 = \sqrt{R_2^2 + X_2^2}$	$Z_{2s} = R_2 + jsX_2 = \sqrt{R_2^2 + (sX_2^2)}$
Rotor current per phase	$I_2 = \frac{E_2}{Z_{20}}$	$I_{2s} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2^2)}} = \frac{E_2}{\sqrt{\left[\frac{R_2}{s}\right]^2 + X_2^2}}$
Rotor power factor	$\cos\phi_2 = \frac{R_2}{Z_{20}} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$	$\cos\phi_{2s} = \frac{R_2}{\sqrt{R_2^2 + (sX_2^2)}} = \frac{R_2}{\sqrt{\left[\frac{R_2}{s}\right]^2 + X_2^2}}$

*Rotor resistance remains the same in both the conditions.

Example .5 A 3-phase, 50 Hz, 8-pole induction motor has a slip of 6%. Find (i) speed of the motor and (ii) frequency of the rotor emf. If the rotor has a resistance of 2 Ω and standstill reactance of 4 Ω, find the power factor at standstill position, and at a speed of 650 rpm.

Solution: $N_s = \frac{120f}{P} = 120 \times \frac{50}{8} = 750$ rpm

$s = 4.5\% = 0.04$ pu

Speed of the motor, $N = (1 - s)\, N_s = (1 - 0.06) \times 750 = 705$ rpm

Frequency of the rotor emf, $f_r = sf = 0.06 \times 50 = 3$ Hz

(i) $R_2 = 2\ \Omega$ and $X_2 = 4\ \Omega$

Rotor importance at stand still, $Z_{20} = R_2 + jX_2 = 2 + j4 = 447\ \angle 63.43°$

Power factor at standstill, $\cos\phi_2 = \cos 63.43° = 0.44$ lagging

(ii) Slip at the rpm of 650 rpm, $s_1 = \frac{N_s - N}{N_s} = \frac{750 - 650}{750} = 0.13$

Rotor impedance at s_1 is given as: $Z_{2s1} = R_2 + j_{s1}X_2 = 2 + j0.13 \times 4 = 2 + j0.52\ \Omega$

$\therefore$ $$Z_{2s1} = 2.06\ \angle 14.57°$$

Power factor at 650 rpm, cos 14.57 = 0.96

Example .6 When a 3-phase, slip ring motor is applied with normal stator voltage, it gives 70 V across the slip ring at rest. The rotor is star connected and its impedance is $(1 + j8)$ per phase. Calculate the current when the machine is (i) at standstill with the slip ring connected with a star-connected starter with a phase impedance of $(5 + j4)\ \Omega$, and (ii) running normally with a slip of 4%.

Solution: E_2 = Rotor induced emf per phase at standstill $= \dfrac{70}{\sqrt{3}} = 40.41$ V

Total impedance of the motor = Rotor stator impedance = Stator impedance

$\therefore$ $$Z_{20} = (1 + j8) + (5 + j4) = 6 + j12 = 13.41\ \angle 63.43°$$

(i) Rotor current at stand still, $I_2 = \dfrac{E_2}{Z_{20}} = \dfrac{40.41\ \angle 0°}{13.41\ \angle 63.43°} = 3.013\ \angle{-63.43°}$ A

(ii) Now, slip, $s = 4\% = 0.04$ pu

$\therefore$ Rotor current at slip of 4%, $I_{2s} = \dfrac{sE_2}{Z_{2s}} = \dfrac{0.04 \times 40.41}{1 + j8 \times 0.04} = \dfrac{1.61}{1 + j0.32} = \dfrac{1.61}{1.04\ \angle 17.74°}$

$$I_{2s} = 1.54\ \angle{-17.74°}$$

Example .7 The induced emf between the slip ring terminals of a 3-phase induction motor is 80 V, when the motor is supplied with normal supply voltage. The value of rotor impedance is $(0.006 + j0.1)\ \Omega$ per phase. Find the rotor current and power factor at (i) 5% slip and (ii) 100% slip.

Solution: Rotor emf per phase, $E_2 = \dfrac{80}{\sqrt{3}} = 46.18$ V

(i) At slip 5% = 0.04 pu, $sE_2 = 0.05 \times 46.18 = 2.30$ V

$$Z_{2s} = \sqrt{R_2^2 + sX_2^2} = \sqrt{0.006^2 + 0.1^2} = 0.100\ \Omega$$

Rotor current at 5% slip, $I_2 = \dfrac{sE_2}{Z_{2s}} = \dfrac{2.30}{0.100} = 23.09$ A

$$\cos\phi_{2s} = \frac{R_2}{Z_{2s}} = \frac{0.006}{0.100} = 0.06,\ \phi_{rs} = 86.56$$

(ii) When slip = 1, i.e., at standstill

$$sE_2 = 1 \times 46.18 = 46.18\ \text{V}$$

$$Z_{2s} = \sqrt{R_2^2 + sX_2^2} = \sqrt{0.006^2 + 1^2} = 1.00\ \Omega$$

Rotor current at 100% slip, $I_2 = \dfrac{sE_2}{Z_{2s}} = \dfrac{46.18}{1.00} = 46.18$ A

$$\cos\phi_{2s} = \frac{R_2}{Z_{2s}} = \frac{0.006}{1.00} = 0.006$$

Example .8 When a 3-phase, 6-pole, 50 Hz, induction motor is running on full load, the frequency of the rotor is found to be 2 Hz. Find the speed of the motor.

Solution:

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$f_r = sf$$

∴ $$s = \frac{f_r}{f} = \frac{2}{50} = 0.04 = 4\%$$

$$N = (1 - s)\, N_s = (1 - 0.04) \times 1000 = 960 \text{ rpm}$$

8 TORQUE

The main principle of the motor is to convert electrical power into mechanical power. So, induced torque, or electromechanical torque, or developed torque of the induction motor depends on the rotor current, rotor power factor and rotating flux. So, the torque is given by

$$T_{ind} \propto \phi I_2 \cos\phi_2$$

where ϕ = Rotating flux of stator
I_2 = Rotor current per phase
$\cos\phi_2$ = Rotor power factor

Now, rotor emf per phase at stand still, $E_2 \propto \phi$

∴ $T_{ind} \propto E_2 I_2 \cos\phi_2$ or $T_{ind} = kE_2 I_2 \cos\phi_2$ where k is constant.

Putting the value of I_2 and $\cos\phi_2$, we can get the torque during running condition.

$$T_{ind} = kE_2 \times \frac{sE_2}{\sqrt{R_2^2 + s^2X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + s^2X_2^2}}$$

Correct the equation (8.1) as under:

∴ $$T_{ind} = \frac{k s R_2 R_2^2}{R_2^2 + (sX_2)^2} \text{ N} - \text{m} \qquad (8.1)$$

8.1 Starting Torque

The name itself indicates that the torque developed by the motor during the starting period is called starting torque. When the induction motor is stationary, the slip is 1. Therefore, starting torque of the motor is obtained by putting the value of slip = 1 in the torque Eqn. (8.1).

$$\therefore \qquad \text{Starting torque } T_{st} = \frac{kR_2E_2^2}{R_2^2 + X_2^2} \qquad (8.2)$$

For getting maximum starting torque, $\frac{dT_{st}}{dR_2} = 0$. Solving the Eqn. (8.2), finally we get $R_2 = X_2$. Generally, the rotor resistance is not more than 1 to 2% of its leakage reactance for higher efficiency. To get higher starting torque, extra resistance is added in to the rotor circuit at the starting time and cut slowly as motor get speed.

So, we can say that maximum starting torque is produced by the induction motor when the rotor resistance is equal to the rotor standstill reactance.

8.2 Condition for Maximum Running Torque

The torque developed by the motor during the running condition is called as running torque. When the motor is running, the torque is given by the Eqn. (8.1), i.e., $T_{RUN} = \frac{ksR_2E_2^2}{R_2^2 + s^2X_2^2}$.

Now the torque will be maximum if $\frac{sR_2}{R_2^2 + s^2X_2^2}$ or $\frac{R_2}{\frac{R_2^2}{s} + sX_2^2}$ or $\frac{R_2}{\left\{\frac{R_2}{\sqrt{s}} - X_2\sqrt{s}\right\}^2 + 2R_2X_2}$ is zero.

The torque will be maximum when the right hand side of the equation is maximum which is possible when $\frac{R_2}{\sqrt{s}} - X_2 s = 0$

$$\therefore \qquad R_2 = sX_2 \quad \text{or} \quad s = \frac{R_2}{X_2}$$

Hence, the induced torque is maximum when the rotor resistance per phase is equal to the rotor reactance per phase under running condition.

Putting the $R_2 = sX_2$ in the torque Eqn. (8.1), we get

$$T_{max} = \frac{ks\,R_2E_2^2}{R_2^2 + R_2^2} = \frac{ksE_2^2}{2R_2} = \frac{ksE_2^2}{2sX_2} \qquad (8.3)$$

Equation (8.3) shows that the maximum torque is independent of the rotor resistance. If s_{Max} is the value of the slip at which the maximum torque is obtained, then $s_{Max} = \frac{R_2}{X_2}$

Hence, the speed of the motor at maximum torque is given as $N_m = (1 - s_{max}) N_s$

Facts on Torque in a Nutshell

(i) Maximum torque is not dependent on the rotor resistance.

(ii) Maximum torque is inversely proportional to the standstill reactance of the rotor. So, to get maximum torque, X_2 and therefore the inductance of the rotor should be kept as small as possible.

(iii) The slip s_{Max} at which the maximum torque occurs depends on the rotor resistance. So, by changing the rotor resistance of the rotor, maximum torque can be achieved at any desired slip. Now the rotor resistance in the rotor can be changed only in slip ring motor. To get maximum torque at standstill, the rotor resistance must be high, and during running condition, it must be low.

8.3 Relation Between Starting Torque and Maximum Torque

From the equations of starting torque (8.2) and maximum torque (8.3), we can get

$$\frac{T_{st}}{T_{max}} = \frac{kR_2E_2^2}{R_2^2 + X_2^2} \times \frac{2sX_2}{ksE_2^2} = \frac{2R_2X_2}{R_2^2 + X_2^2}$$

Dividing the numerator and denominator by X_2^2, we get

$$\frac{T_{st}}{T_{max}} = \frac{2R_2X_2/X_2^2}{\frac{R_2^2}{X_2^2} + \frac{X_2^2}{X_2^2}} = \frac{2R_2/X_2}{\left(\frac{R_2}{X_2}\right)^2 + 1}$$

Now, we know that the slip at which maximum torque occurs is given as $s_{max} = \frac{R_2}{X_2}$
Hence, we can get

$$\frac{T_{st}}{T_{max}} = \frac{2s_{max}}{s_{max}^2 + 1} \qquad (\text{¯ } 8.4)$$

8.4 Relation between Full Load Torque and Maximum Torque

In a manner similar to the above, we can get the relation between the full load torque and the maximum torque of the induction motor as under.

$$T_f = \frac{ksR_2E_2^2}{R_2^2 + s^2X_2^2}$$

$$\frac{T_f}{T_{max}} = \frac{ksR_2E_2^2}{R_2^2 + (sX_2)^2} \times \frac{2sX_2}{ksE_2^2} = \frac{sR_2\,2\,X_2}{R_2^2 + (sX_2)^2}$$

Dividing the numerator and denominator by X_2^2, we get

$$\frac{T_{ind}}{T_{max}} = \frac{sR_2^2X_2/X_2^2}{R_2^2 + (sX_2)^2/X_2^2} = \frac{2s\ s_{max}}{s_{max}^2 + s^2}$$

Hence, the relation between full load torque and maximum torque is

$$\frac{T_{ind}}{T_{max}} = \frac{2ss_{max}}{s_{max}^2 + s^2} \qquad (8.5)$$

8.5 Starting Torque of Squirrel Cage Motor

We know that the resistance of the squirrel cage motor is fixed and very small compared to its reactance because at the standstill the frequency of the rotor current is equal to the supply frequency. So, the rotor current I_2 is large and lags by a wide angle behind E_2 so the starting torque per ampere is very poor. It is hardly 1.5 times the full load torque and starting current is 5 to 7 times the full load current. Hence such motors are not applicable where high starting torque is required.

8.6 Starting Torque of Slip Ring Motor

In a slip ring motor, the torque can be increased by improving its power factor by adding extra resistance in the rotor. As the motor gains speed, this resistance is slowly cut down. This extra resistance increases the rotor impedance so the rotor current is reduced.

.9 POWER

We know that rotor current per phase is given by, $I_2 = \dfrac{sE_2^2}{\sqrt{R_2^2 + (sX_2^2)}} = \dfrac{E_2}{\sqrt{\left[\dfrac{R_2}{s}\right]^2 + X_2^2}}$ which can be presented by simple series circuit as shown in Fig. 9.

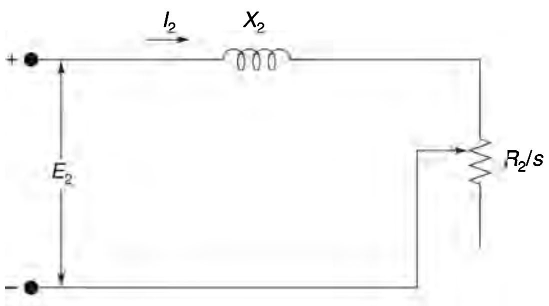


Fig. .9

From Fig. .9, it is clear that per phase input power (gross) to the rotor $P_{AG} = E_2I_2 \cos\phi_2$.

where
$$\cos\phi_2 = \frac{R_2/s}{\sqrt{\left\{\dfrac{R_2}{s}\right\}^2 + X_2^2}}$$

$$\therefore \qquad P_{AG} = \frac{E_2}{\sqrt{\left[\frac{R_2}{s}\right]^2 + X_2^2}} \times I_2 \frac{R_2}{s} = I_2^2 \frac{R_2}{s}$$

$$\text{Air gap power, } P_{AG} = I_2^2 \frac{R_2}{s} \qquad (\quad 9.1)$$

So, from Fig. 9, we can say that per phase power input to the rotor is equal to the $I_2^2 \frac{R_2}{s}$ because no power is consumed in reactance X_2.

P_{AG} is the power transferred from stator to the rotor via air gap so this power is also called air gap power. This power may be written as $P_{AG} = I_2^2 \frac{R_2}{s} = I_2^2 R_2 \left[\frac{1-s}{s}\right]$

$$\therefore \qquad \text{Rotor copper loss} = s P_{AG} \qquad (\quad 9.2)$$

Thus the copper loss is equal to the slip times rotor input (air gap power). This air gap power is also called as slip power. It is the portion of the air gap power which is not converted in to the mechanical power.

Now, P_{AG} = Rotor ohmic loss + Internal mechanical power developed in rotor (P_{mech})

$\therefore$ Rotor ohmic loss = Power transferred from stator to rotor (P_{AG}) – Mechanical power developed by the rotor (P_{mech})

$$P_{mech} = P_{AG} - \text{Rotor copper loss} = P_{AG} - sP_{AG} = P_{AG}(1-s) = I_2^2 R_2 \left\{\frac{1-s}{s}\right\}$$

or,

$$\text{Rotor copper loss} = P_{mech} \left\{\frac{s}{1-s}\right\} = s P_{AG}$$

Internal (gross) torque developed per phase is given by

$$T_{ind} = \frac{\text{Internal mechanical power developed in the rotor}}{\text{Rotor speed in mechanical radian per second}}$$

$$T_{ind} = \frac{P_{mech}}{\omega_r} = \frac{P_{AG}(1-s)}{\omega_s(1-s)} = \frac{P_{AG}}{\omega_s} \qquad (\quad 9.3)$$

So from the above discussion, if once the air gap power is found, three more quantities can be found as given below:

$$\text{Rotor copper loss} = s P_{AG}$$

$$P_{mech} = P_{AG}(1-s)$$

$$T_{ind} = \frac{P_{AG}}{\omega_s}$$

Equation (9.3) is more important because it gives the induced or developed torque directly in terms of air gap power and synchronous speed is constant and not depends on loading condition of the motor, the torque can be obtained directly from the air gap power. Since the torque is given by the above equation (9.3), the air gap power is sometime called "the torque in synchronous watts". Synchronous watt is the torque that develops power of 1 watt when the machine is running at synchronous speed.

Example .9 A 3-phase, 6-pole induction motor has rotor resistance of 0.05 Ω per phase. The maximum torque occurs at a speed of 900 rpm. Find the starting torque as a percentage of maximum torque.

Solution:

$$N_s = \frac{120f}{p} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Now,

$$N_M = (1 - s_{max})\, N_s$$

$$900 = N_s - N_s\, S_{max}$$

∴

$$S_{max} = \frac{N_s - 900}{N_s} = \frac{1000 - 900}{1000} = 0.1$$

$$\frac{T_{st}}{T_{max}} = \frac{2\, s_{max}}{1 + s_{max}^2} = \frac{2 \times 0.1}{1 + 0.1^2} = \frac{0.2}{1.01} = 0.198$$

$$T_{st} = 0.198\, T_{max} = 19.38\%\, T_{max}$$

Example .10 An 8-pole, 3-phase, 50 Hz, induction motor has a full load slip of 4%. The rotor phase resistance is 0.3 Ω and standstill reactance is 1.1 Ω. Find the ratio of maximum torque to the full load torque of the motor and also find the speed at which the maximum torque is produced.

Solution: Slip corresponding to maximum torque $S_M = \frac{R_2}{X_2} = \frac{0.3}{1.1} = 0.27$

$$\frac{T_{max}}{T_{ind}} = s^2 + \frac{s_{max}^2}{2 s\, s_{max}} = \frac{0.04^2 + 0.27^2}{2 \times 0.04 \times 0.27} = \frac{0.074}{0.0216} = 3.42$$

$$N_s = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

Speed at maximum torque produced, $N_M = (1 - s_{max})\, N_s = (1 - 0.27) \times 750$

$$N_M = 547 \text{ rpm}$$

Example .11 The input power to 3-phase, 50 Hz induction motor is 50 kW. The total stator loss is 2 kW. Calculate the mechanical power developed by the motor and the rotor copper loss per phase if the slip is of 4%.

Solution: Stator input, P_i = 50 kW, s = 4% = 0.04 pu

Stator loss = 1 kW Stator output = 50 – 1 = 49 kW = Rotor input

Total rotor copper loss = s × Rotor input = 0.04 × 49 = 1.96 kW

Rotor copper loss per phase = $\frac{1.96}{3}$ = 0.65 kW

Mechanical power developed = Rotor input – Rotor copper loss

= 49 – 0.65 = 48.35 kW

Example .12 An 8-pole, 50 Hz, 3-phase induction motor running on full load develops a useful torque of 150 Nm at a rotor frequency of 2 Hz. Find the shaft output power. If the mechanical torque lost in the form of friction is 10 Nm, calculate (i) rotor copper loss, (ii) input power of the motor and (iii) the efficiency of the motor. The total stator loss is 800 W.

Solution:

$$N_s = \frac{120 \times 50}{8} = 750$$

$$s = \frac{f_r}{f} = \frac{2}{50} = 0.04 = 4\%$$

$$N = (1 - s)N_s = (1 - 0.04) \times 750 = 720 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 720}{60} = 75.39 \frac{\text{rad}}{\text{sec}}$$

Shaft power output = 150 × 75.39 = 11.309 kW

$$P_{\text{mech}} = (150 + 10) \times 75.39 = 12.062 \text{ kW}$$

(i) Rotor copper loss, $P_{R\text{copper}} = \left(\frac{s}{1-s}\right) P_{\text{mech}} = \frac{0.04}{1 - 0.04} \times 12062 = 0.50$ kW

(ii) Input of the motor, $P_{\text{in}} = P_{\text{mech}} + P_{R\text{copper}} + P_{S\text{copper}} = 12.062 + 0.50 + 0.800$

$P_{\text{in}} = 13.362$ kW

(iii) Efficiency = $\frac{\text{Output}}{\text{Input}} = \frac{11.309}{13.362} = 0.8463$ pu = 84.63%

Example .13 The input power to a rotor of 415 V, 50 Hz, 4-pole, induction motor is 75 kW. The rotor emf has been observed to be making 100 rpm. Find (i) slip, (ii) rotor speed, (iii) mechanical power developed, (iv) rotor copper loss per phase and (v) rotor resistance per phase if the rotor current is 60 A.

Solution: $f = 50$ Hz and $f_r = \frac{100}{60} = 1.66$ Hz

(i) $s = \frac{f_r}{f} = \frac{1.66}{50} = 0.03$ pu

$$N_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

(ii) $N = (1 - s)N_s = (1 - 0.03) \times 1500$ rpm = 1455 rpm

(iii) P_{mech} = Rotor input – Rotor copper loss (Rotor copper loss – s × Rotor input)

$$P_{mech} = 75 \times 1000 - 0.03 \times 75 \times 1000 - 72750 = 72.75 \text{ kW} = \frac{72750}{746} = 97.54 \text{ HP}$$

(iv) Rotor copper loss per phase = $s \times 75000 = 0.03 \times 75000 = 2250$ W

(v) Rotor resistance per phase = R_2 = (Rotor copper loss per phase)/I^2

$$R_2 = \frac{2250}{60^2} = 1.6\ \Omega$$

Example .14 A 415 V, 8-pole, 50 Hz, induction motor of 25 kW power including mechanical losses is running at 720 rpm at the power factor 0.8 lagging. Find (i) the slip, (ii) the rotor copper loss, (iii) total input if the stator loss is 1500 W, (iv) line current and (v) the rotor current frequency.

Solution:

$$N_s = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

(i) Slip, $s = \frac{N_s - N}{N_s} = \frac{750 - 720}{750} = 0.04$ pu = 4%

(ii) $P_{Rcopper} = s \times$ Rotor Input
= $s \times$ (Mechanical power developed + Rotor copper loss)

$$P_{Rcopper} = s(P_{mech} + P_{Rcopper})$$

$$P_{Rcopper} = (1 - s) = s\ P_{mech}$$

$$P_{Rcopper} = \frac{s P_{mech}}{1 - s} = \frac{0.04 \times 25 \times 1000}{1 - 0.04} = 1041.66 \text{ W}$$

(iii) Total stator Input = Rotor input power + Stator loss

$$= \frac{1}{s} \times \text{Rotor copper loss} = \text{Stator loss}$$

$$= \frac{1041.66}{0.04} + 1500 = 27.54 \text{ kW}$$

(iv) Line current = $\frac{27540}{\sqrt{3} \times 415 \times 0.8} = 47.89$ A

(v) $f_r = sf = 0.04 \times 50 = 2$ Hz

Example .15 The resistance of the rotor and standstill reactance per phase of a 3-phase slip ring induction motor is 0.004 and 0.3 Ω, respectively. Find the value of external resistance per phase to be inserted in the rotor so that maximum torque can be produced.

Solution: Let r be the value of the external rotor resistance in ohms

$$R_2 = 0.004 + r$$

The starting torque will be maximum when $R_2 \times X_2$

$$0.004 + r = 0.3$$

$$\therefore \quad r = 0.3 - 0.004 = 0.296\ \Omega$$

Example .16 A 4-pole, 3-phase, 50 Hz induction motor is developing a maximum torque of 35 Nm at 1460 rpm. Find the torque developed by the motor at 3% slip. The rotor resistance per phase is 0.5 Ω.

Solution: $N_s = \dfrac{120 \times 50}{4} = 1500$ rpm

Speed at maximum torque, $N_M = 1450$ rpm

Slip at maximum torque $s_{max} = \dfrac{N_s - N_M}{N_s} = \dfrac{1500 - 1460}{1500} = 0.02 = 2\%$

Now, $s_{max} = \dfrac{R_2}{x_2}$, $\therefore X_2 = \dfrac{0.5}{0.02} = 25\ \Omega$

If T is torque at slip, s $\dfrac{T}{T_{max}} = \dfrac{2 s s_{max}}{s^2 + s_{max}^2}$

$$\therefore\ T = \frac{2 \times 0.03 + 0.02 \times 35}{0.03^2 + 0.02^2} = \frac{0.042}{0.0013} = 32.30 \text{ Nm}$$

10 EFFECT OF CHANGE IN SUPPLY VOLTAGE ON THE STARTING TORQUE

We know that the starting torque is given by, $T_{st} = \dfrac{kR_2E_2^2}{R_2^2 + X_2^2}$

When the rotor is stationary, induced emf in the rotor is $E_2 \propto \phi \propto V$.

$$\therefore \qquad T_{st} = \frac{k' R_2 V_2^2}{R_2^2 + X_2^2} \qquad \text{where } k' \text{ is other constant}$$

$$\therefore \qquad \boxed{T_{st} \propto V^2} \qquad (10.1)$$

Hence, we can say that starting torque is proportional to the square of the supply voltage.

If the motor is running at slip s then torque of the motor is given by

$$T_{ind} = \frac{ksR_2E_2^2}{R_2^2 + s^2X_2^2} = \frac{k'sR_2V_2^2}{R_2^2 + s^2X_2^2} \quad \text{where } k' \text{ is other constant}$$

Now slip at full load is very low, so $s^2X_2^2$ can be neglected with compared to the R_2^2.

$$\therefore \qquad T_{ind} = sV^2$$

Hence, when the supply voltage is changed, the torque under running condition also changes. So, to maintain the same torque, the slip should be increased or the speed decreased.

11 WINDING EMF

Let V_1 be the per phase voltage applied to the stator.

T_1 as the number of stator winding turns in series per phase.

T_2 the number of rotor turns in series per phase.

ϕ as flux per pole produced by the stator.

E_1 the stator induced emf per phase.

E_2 the rotor emf when the rotor is standstill.

E_{2s} as emf induced in the rotor when the rotor is running at slip s.

R_1 as stator resistance per phase.

R_2 the rotor resistance per phase.

L_2 is the rotor inductance per phase at standstill due to leakage flux.

X_2 the leakage reactance of the rotor per phase when the rotor is standstill.

f as stator frequency.

f_r as frequency of the induced emf in the rotor at a slip s.

sX_2 the leakage reactance of the rotor per phase at the slip s.

k_{d1} the distribution factor of stator.

k_{d2} the distribution factor of rotor.

k_{c1} the coil span factor of stator.

k_{c2} the coil span factor of rotor.

$\therefore$ Stator induced emf per phase $E_1 = 4.44\ k_{c1} k_{d1} f \phi T_1$

Induced emf in the rotor per phase when it is at standstill

$$E_2 = 4.44\ k_{c2} k_{d2} f \phi T_2$$

Induced emf in the rotor per phase when it is running at slip s

$$E_{2s} = 4.44\ k_{c2} k_{d2} f_r \phi T_2 = sE_2$$

Now, let $k_{c1} k_{d1} = k_{w1}$ = Winding fector of the stator.

$k_{c2}, k_{d2} = k_{w2}$ = Winding factor of the rotor

$$\therefore \quad E_1 = 4.44\ k_{w1} f \phi T_1$$

$$E_{2s} = 4.44\ k_{w2} f_r \phi T_2$$

Now suppose, $T_{e1} = k_{w1} T_1$ and $T_{e2} = k_{w2} T_2$

$$\frac{E_1}{E_2} = \frac{k_{w1} T_1}{k_{w2} T_2} = T_{eff}$$

where T_{e1} and T_{e2} are the effective stator and rotor turns per phase.

T_{eff} = Effective turn ratio of an induction motor.

$$\frac{I_2'}{I_2} = \frac{T_{e1}}{T_{e2}} = \frac{1}{T_{eff}}$$

From the above equation, it is clear that the ratio between stator emf and rotor emf is constant at standstill. Thus, the induction motor behaves like a transformer. Here, it is to be noted that the factors for stator and rotor windings are not same because the number of stator and rotor turns are not same.

12 EQUIVALENT CIRCUIT

We know that the induction motor is considered as a transformer in which energy is transferred from stator to rotor with change in frequency. This energy is transferred via the air gap. When the motor is stationary, it acts as a simple transformer with an air gap and secondary is short circuited. At standstill, the frequency of the rotor-induced emf is the same as the supply frequency. At any value of slip *s*, the rotor current reacts on the stator winding at the stator frequency because the rotating magnetic field and rotor field are stationary with respect to each other.

12.1 Stator Circuit

Thus, induction motor may be viewed as a transformer with an air gap and a variable resistance in the secondary. The performance characteristics of the induction motor can be found by simple network using equivalent circuits. The equivalent circuit of stator model is given in Fig. 10.

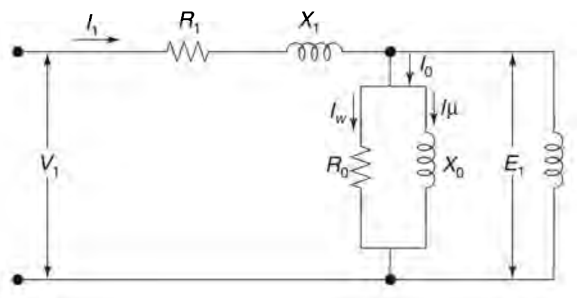


Fig. 10 Equivalent circuit of stator

This stator model has a winding resistance R_1 and leakage reactance X_1. The no load current I_0 simulated by a pure inductive reactor X_0 carrying magnetizing current I_μ and non-inductive resistor R_0 carries core loss current I_ω.

Thus, $$I_0 = I_\mu + I_\omega.$$

The total magnetizing current I_0 in the induction motor is comparatively larger than transformer because air gap has a higher reluctance in the motor. In a transformer the value of I_0 is about 2 to 5% of the full load current while in induction motor it is about 25 to 40% of the full load current.

12.2 Rotor Circuit

When a 3-phase supply is applied to the stator of the induction motor, a voltage is induced in the rotor. This rotor voltage depends on the relative motion between the rotor and the stator field. At standstill, this relative motion is the largest. This is also called locked or blocked rotor condition. If the induced voltage at standstill is E_2, then the induced voltage at any slip is given by

$$sE_2 = \text{Slip} \times \text{Induced voltage at standstill}$$

The rotor resistance is given by R_2 and is independent of the slip. The reactance of the rotor X_2 is dependent on the inductance of the rotor and frequency of the voltage.

Hence, if L_2 is the inductance of the rotor, then $X_2 = 2\pi f_r L_2$. But $f_r = sf$

$\therefore$ $X_2 = 2\pi sf L_2 = s(2\pi f L_2) = sX_2$, where X_2 is standstill reactance of the rotor.

The rotor circuit is shown in Fig. 11.

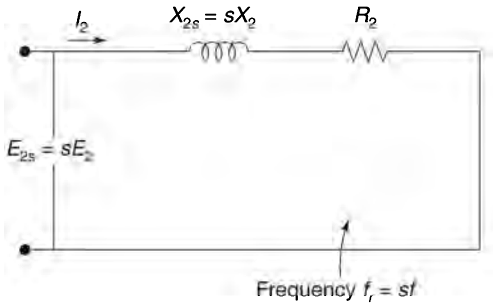


Fig. 11 Equivalent circuit of rotor

The rotor impedance is given by, $Z_{2s} = R_2 + jX_{2s}$

The rotor current
$$I_{2s} = \frac{E_2 s}{Z_{2s}} \qquad (12.1)$$

$$I_{2s} = \frac{sE_2}{R_2 + jsX_2} \qquad (12.2)$$

From the Fig. 11, we can say the I_2 is a slip frequency current produced by sE_2 which is the slip-frequency-induced emf in the rotor. If we divide the Eqn. (.12.2) with the slip s, we get

$$I_{2s} = \frac{E_2}{\frac{R_2}{s} + jX_2} \qquad (12.3)$$

So, if we draw the rotor circuit as per Eqn. (12.3), it will be as shown in Fig. 12. Here, it is noted that the magnitude and the phase angle of the current I_2 is same. But there is a major difference in Eqn. (.12.2) and (12.3). In Eqn. (12.3), the rotor current I_2 is produced by a constant line frequency voltage E_2 and while in Eqn. (12.2) it is produced by a slip frequency voltage.

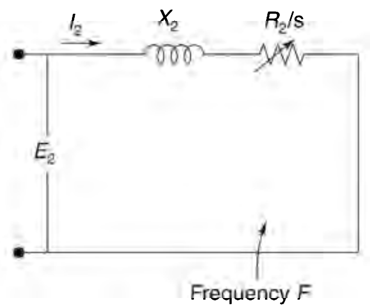


Fig. 12

One more thing is noted that in Fig. 11, in the rotor circuit resistance R_2 is constant while leakage reactance sX_2

is variable. On the other hand, in Fig. 12, leakage reactance X_2 is constant and resistance $\frac{R_2}{s}$ is variable.

Here, Eqn. (12.3) is an important one, so it should be understood very clearly. It indicates the secondary circuit of a imaginary transformer whose voltage ratio is constant with same frequency at both sides. The current drawn by the imaginary stationary rotor is the same as that of the rotating rotor and producing the same mmf wave. By this concept, it is possible to transfer the rotor (secondary) impedance to the primary (stator) side.

It should be clearly understood that when the rotor current and voltages are reflected to the stator, their frequency also changes to the stator frequency.

12.3 COMPLETE CIRCUIT MODEL REFERRED TO STATOR

To get the complete per phase equivalent circuit of the induction motor, it is required to refer the rotor circuit over to the stator circuit. If we talk about the transformer, the voltage, current and impedances on the secondary side can be transferred to the primary side by turns ratio. It is done similarly in the case of induction motor.

If a is the effective turn ratio of the induction motor.

R'_2 is the rotor resistance per phase referred to the stator

X'_2 is rotor reactance per phase referred to the stator

$$\frac{E_2}{T_2} = \frac{E'_2}{T_1}$$

∴ $$E'_2 = \frac{T_1}{T_2} E_2 = a E_2 = E_1$$

Similarly, $$I'_2 = \frac{I_2}{a}$$

$$Z'_2 = a\left(\frac{R_2}{s} + jX_2\right)$$

$$R'_2 = a^2 R_2$$

$$X'_2 = a^2 X_2$$

The complete circuit model is shown in Fig. .13. This model is identical with the 2-winding transformer.

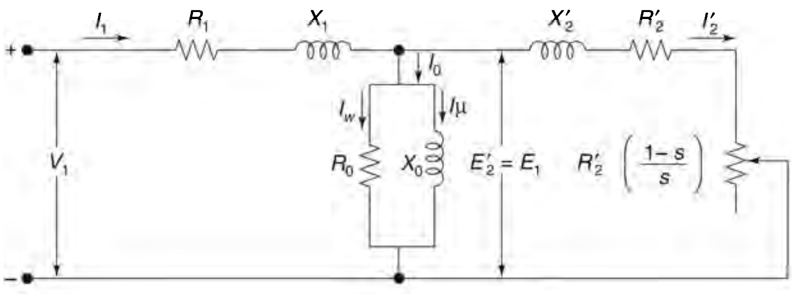


Fig. .13 Complete circuit model

12.4 Approximate Equivalent Circuit

The complete circuit (Fig. 13) is further simplified by shifting the shunt impedance branches R_0 and X_0 to the input terminal for easy calculation as shown in Fig. 14. Here it is assumed that $V_1 \cong E_1 \cong E'_2$. This circuit is called approximate equivalent per phase circuit of the induction motor. In this circuit, only the resistance which represents the developed mechanical power by the rotor depends on the slip, while all other quantities are constant. This circuit is most popular and is a standard for all performance calculations of an induction motor.

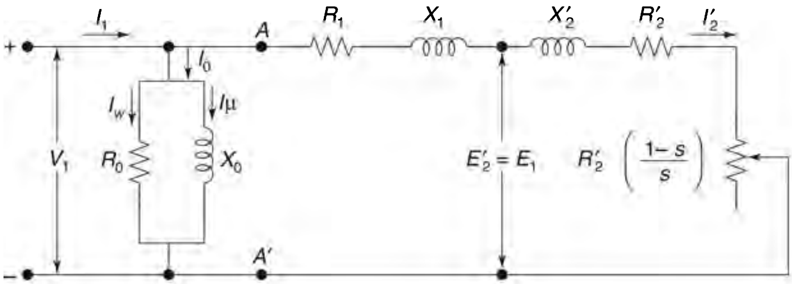


Fig. 14 Approximate equivalent circuit

From Fig. 14, the following equations at any slip can be obtained:

Impedance at $$AA' = Z_{AA'} = \left(R_1 + \frac{R'_2}{s}\right) + j(X_1 + X'_2)$$

$\therefore$ $$I'_2 = \frac{V_1}{Z_{AA'}} = \frac{V_1}{\left(R_1 + \frac{R'_2}{s}\right) + j(X_1 + X'_2)}$$

$\therefore$ $$I'_2 = \frac{V_1}{\sqrt{\left(R_1 + \frac{R'_2}{s}\right)^2 + (X_1 + X'_2)^2}}$$

$\therefore$ $$I'_2 = I'_2 \angle -\phi_2 = I'_2 \cos\phi_2 - jI'_2 \sin\phi_2$$

where $$\tan\phi_2 = \frac{X_1 + X'_2}{R_1 + \frac{R'_2}{s}} \quad \text{and} \quad \cos\phi_2 - \frac{R_1 + \frac{R'_2}{s}}{Z_{AA'}}$$

No load current, $$I_0 = I_\omega + I_\mu = \frac{V_1}{R_0} + \frac{V_1}{jX_0} = V_1\left(\frac{1}{R_0} - j\frac{1}{X_0}\right)$$

Total stator current, $$I_1 = I_0 + I'_2$$

Total core losses, $$P_{core} = 3V_1 I_0 \cos\phi_0$$

Stator input $= 3V_1 I_1 \cos\phi_1 = 3V_1 I'_2 \cos\phi_2 + P_{core} = 3\, I'^2_2 \left(R_1 + \frac{R'_2}{s}\right) + P_{core}$

Air gap power per phase, $P_{AG} = V_1 I'_2 \cos\phi_2 = I'^2_2 \frac{R'_2}{s} = \frac{V_1^2 (R'_2/s)}{\left(R_1 + \frac{R'_2}{s}\right)^2 + (X_1 + X'_2)^2}$

Induced torque, $T_{ind} = \frac{P_{AG}}{\omega_s}$

12.5 Separation of Mechanical Losses from Rotor Copper Loss in the Circuit Model

In the per-phase complete equivalent circuit, the resistance R'_2/s consumes the total rotor input (air gap power). So, air gap power is $P_{AG} = 3I'^2_2 \frac{R'_2}{s}$.

The actual copper losses in the rotor are given by $P_{Rcopper} = 3\, I'^2_2\, R'_2$

Mechanical power developed $P_{mech} = P_{AG} - P_{Rcopper} = 3I'^2_2 \frac{R'_2}{s} - 3I'^2_2 R'_2$

$$\therefore \quad P_{mech} = 3I'^2_2 R'_2 \left[\frac{1}{s} - 1\right]$$

$$P_{AG} = P_{Rcopper} + P_{mech} = 3I'^2_2 R'_2 + 3I'^2_2 R'_2\left[\frac{1}{s} - 1\right]$$

$$\therefore \quad P_{AG} = 3I'^2_2 \left[R'_2 + R'_2 \left[\frac{1}{s} - 1\right]\right]$$

Also, $R'_2 + R'_2 \left[\frac{1-s}{s}\right] = R'_2 + \frac{R'_2}{s} - R'_2 = \frac{R'_2}{s}$

It is clear that $\frac{R'_2}{s}$ may be divided into two components. R'_2 gives the rotor copper loss per phase and $R'_2\left[\frac{1-s}{s}\right]$ represents the mechanical power developed. We can say that the variable resistance $\frac{R_2}{s}$ shown in Fig. 12 may be replaced by the actual rotor winding resistance R_2 and a variable resistance R_{mech} which gives the mechanical shaft load.

$$R_{mech} = R_2\left(\frac{1}{s} - 1\right) \qquad (12.4)$$

This equation is useful because any mechanical load can be represented by a resistance in the equivalent circuit. The modified per phase circuit is given in Fig. 15 using this expression.

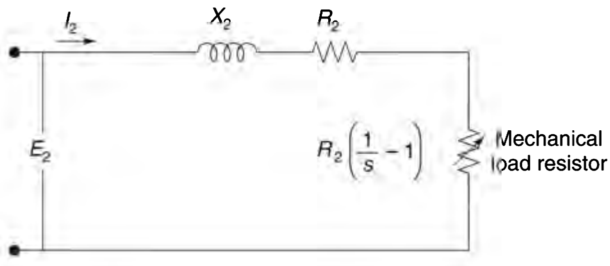


Fig. 15

Example .17 A 415 V, 3-phase, 6-pole, 50 Hz, star-connected induction motor has the following parameters per phase referred to the stator winding: $R_1 = 0.15\ \Omega$, $X_1 = 0.45\ \Omega$, $R_2 = 0.12\ \Omega$ $X_2 = 0.45\ \Omega$ and $X_0 = 28.5\ \Omega$. Find the stator current, motor speed, output torque, and efficiency when the motor is operating at rated voltage and frequency at slip of 4%. Take constant loss as 500 W.

Solution: The equivalent circuit is shown as under in Fig. ¯ 16.

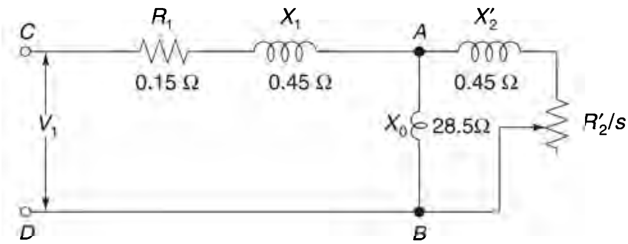


Fig. 16

$$Z_{AB} = \frac{(R_2'/s + jZ_2)\ JX_0}{R_2'/s + j\ (X_2 + X_0)} = \frac{(0.12/0.04 + j0.45)\ (j28.5)}{3 + j28.95} = 2.87 + j0.738$$

$$Z_{CD} = R_1 + jX_1 + Z_{AB} = 0.15 + j0.45 + 2.87 + j0.738 = 3.02 + j1.188$$

$$Z_{CD} = 3.24\ \angle 21.47°$$

$$\text{Stator current} = \frac{415/\sqrt{3}}{3.24\ \angle 21.47°} = \frac{239.60\ \angle 0°}{3.24\ \angle 21.47°} = 73.95\ \angle -21.47°$$

Power factor, cos 21.47° = 0.93 lagging

Motor input, $P_{in} = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 415 \times 73.95 \times 0.93 = 49.434$ kW

Power transferred via air gap, $P_{AG} = 3 \times I_1^2 \times \frac{R_2'}{s} = 3 \times 73.95^2 \times \frac{0.12}{0.04} = 49.217$ kW

Mechanical power developed, $P_{mech} = (1 - s)\ P_{AG} = (1 - 0.04)\ 49.217 - 47.248$ kW

Output power = P_{mech} – Constant loss = 47.248 – 0.5 = 46.748 kW

$$\text{Net torque or shaft torque} = \frac{\text{Output}}{2\pi N} \times 60 = \frac{46.748}{2 \times \pi \times 1440} \times 60 = 310 \text{ Nm}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{46748}{49434} = 0.9456 \text{ pu} = 94.56\%$$

Example .18 A 500 V, 3-phase, star-connected induction motor has the following impedances per phase referred to the stator winding: stator: (0.5 + *j*1) Ω and rotor: (0.7 + *j*1) Ω and magnetizing branch: (10 + *j*50) Ω. Find (i) maximum torque, (ii) slip at which maximum torque produced.

Solution: The approximate equivalent circuit is shown as under in Fig. .17.

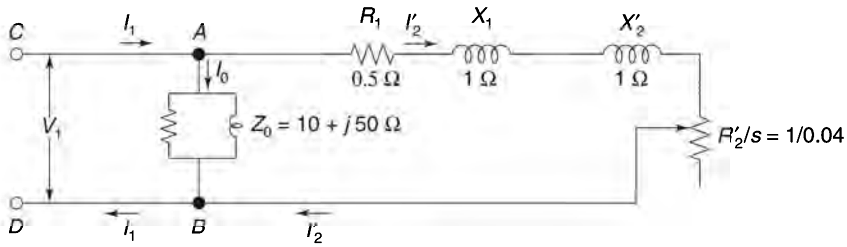


Fig. 17

The slip at which maximum torque is produced, $S_{max} = \dfrac{R'_2}{\sqrt{R_1^2 + (X_2 + X'_2)^2}} = \dfrac{0.7}{\sqrt{0.5^2 + 2^2}}$

$= 33\%$

$$T_{max} = \frac{P_{AG}}{2\pi N_s/60} = \frac{3\dfrac{I'^2_2 R'_2}{s_{max}}}{2\pi N_s/60}$$

$$I'_2 = \frac{V_1}{\sqrt{(R_1 + R'_2)^2 + (X_2 + X'_2)^2}} = \frac{500/\sqrt{3}}{\sqrt{(0.5 + 0.7)^2 + 2^2}} = 53.06 \text{ A}$$

$$T_{max} = \frac{3 \times 53.06^2 \times 0.7/0.33}{2 \times \pi \times 1500/60} = \frac{17915.95}{157.07} = 114.06 \text{ Nm}$$

13 TORQUE SLIP AND ITS CHARACTERISTICS

If the load on an induction motor changes, there is definite effect on induced torque and speed. To understand this, the torque-speed relationship must be examined. First, we will study it for the motor's magnetic field behaviour and then study the general equation for torque as a function of slip will be derived from the equivalent circuit of the motor.

When the motor is at no load, its speed is nearly at the synchronous level. The stator magnetic field, or say mmf of stator, M_{STATOR} in the machine is produced by the magnetizing current I_0 and the magnitude of this current and hence M_{STATOR} is directly proportional to the voltage E_1 of the equivalent circuit. Now, if E_1 is constant, the stator magnetic field is constant. But in the actual machine, E_1 varies as the load changes, because the stator impedance drop varies with load. However, this drop is very small, so E_1, I_0 and hence M_{STATOR} approximately constant with changes in load.

At no load, the slip is small. So, the relative motion between the rotor and magnetic field is small. Thus, the voltage induced in the rotor sE_2 and hence the rotor current is also small. Again, the rotor frequency is small too, so the reactance of the rotor is almost zero. So, the rotor current is almost in phase with rotor induced emf sE_2. This small rotor current produces a small rotor field M_{ROTOR} which is behind the net magnetic field (greater than 90°). Here, note that the stator current must be quite large even at no load because it must supply most of M_{STATOR}. Therefore, induction motors have large no-load current compared to other types of machines.

The induced torque, which keeps the rotor turning is given by

$$T_{ind} = kM_{STATOR} \times M_{ROTOR}$$

Its magnitude is given by $T_{ind} = kM_{STATOR}\, M_{ROTOR} \sin\delta$

Since the rotor field is very small, the induced torque is also small just to overcome the rotational losses.

Now suppose, as the motor is loaded, its slip increases and the speed falls. Since the speed is low, the relative motion between the rotor and stator magnetic field is more and hence more rotor current is produced which in turn increases in M_{ROTOR} Hence, the angle between M_{ROTOR} and rotor current also changes. Since the slip is more, the rotor is more too, resulting in rotor reactance. Therefore, the rotor current now lags behind the rotor voltage. Notice that the rotor current has increased so the angle δ also increases. The increase in M_{ROTOR} tends to increase the torque induced, while increase in the angle δ tends to decrease the induced torque. The first effect is larger than the second effect; hence, the overall induced torque increases to supply the increased load to the motor.

At the point when the term sin δ decreases more than the term M_{ROTOR} increases, there is a limit of pull out torque. After this point, a further increase in load decreases the induced torque and the motor stops.

So, the knowledge of the magnetic field gives approximately torque-speed curve of an induction motor. We know that the magnitude of the induced torque is given by

$$T_{ind} = k\, M_{STATOR}\, M_{ROTOR} \sin\delta$$

Each terms of this equation can be considered separately to derive the overall machine behaviour as under:

(i) The rotor magnetic field M_{ROTOR} is directly proportional to the rotor current (I_2) till the rotor is unsaturated. As the slip increases, the rotor current also increases.

(ii) The stator magnetic field M_{STATOR} in the motor is proportional to E_1 which is almost constant.

(iii) The angle δ between the stator and rotor magnetic field can be expressed in a very useful way. This angle is just equal to the power factor angle of the rotor plus 90 degrees.

$$\delta = \phi_2 + 90°$$

Therefore, $(\sin \phi_2 + 90°) = \cos \phi_2$. This term is the power factor of the rotor.

The plot of the above three terms is shown in Fig. ˜ 18. Since the induced torque is proportional to these three terms, the torque speed curve of the induction motor can be constructed from the multiplication of these three terms as in Fig. 19. This curve can be divided into three parts as under:

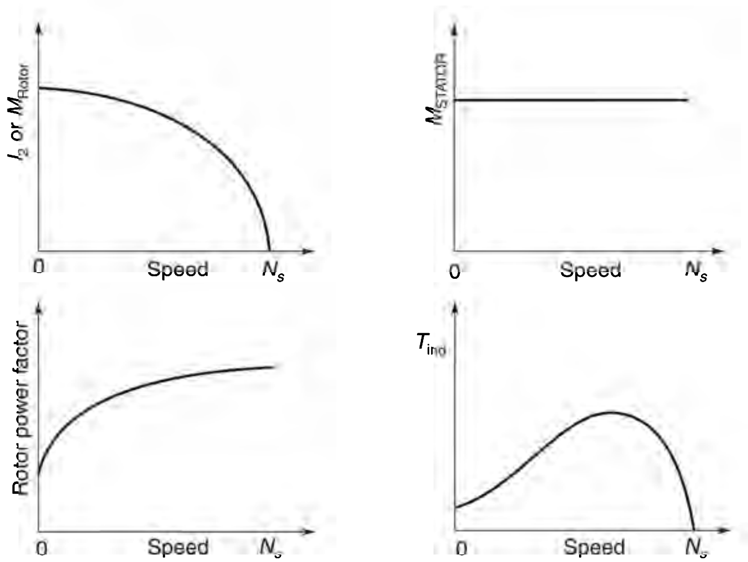


Fig. . 18

(a) The first region is called the low slip region. In this region, as we increase the load, the motor slip linearly increases and at the same time the rotor speed also decreases linearly. In this region, the rotor reactance is negligible, so the rotor power factor is almost unity. The steady state operating range of an induction motor is included in this low slip region.

(b) The second region is the moderate slip region. Here, the rotor frequency is more than before and the rotor reactance is equal to the rotor resistance. Hence, the rotor current cannot increase rapidly as before. The maximum torque or pull out torque occurs at the point where the increment in the rotor current is perfectly balanced by the decreased rotor power factor, while increasing the load.

(c) The third region is called high slip region. Here, the induced torque is decreased while increasing the load.

Now, we will consider a general equation for induced torque in terms of speed. The induced torque in an induction motor is given by

$$T_{ind} = \frac{P_{mech}}{\omega_m}$$

$$T_{ind} = \frac{P_{AG}}{\omega_s}$$

The synchronous speed of the motor is almost constant. So, the air gap power gives the torque of the induction motor. The air gap power is the power crossing from stator to rotor and it is the power absorbed by $\frac{R_2'}{s}$. The total air gap power is given by $P_{AG} = 3\, I'^2_2 \frac{R_2'}{S}$

So, if the rotor current is known, the induced torque can be known too.

A plot of induction motor as a function of slip is plotted as in Fig. 19(a). A plot giving both speeds, above as well as below the normal speed of the motor is shown in Fig. ¯ 19(b).

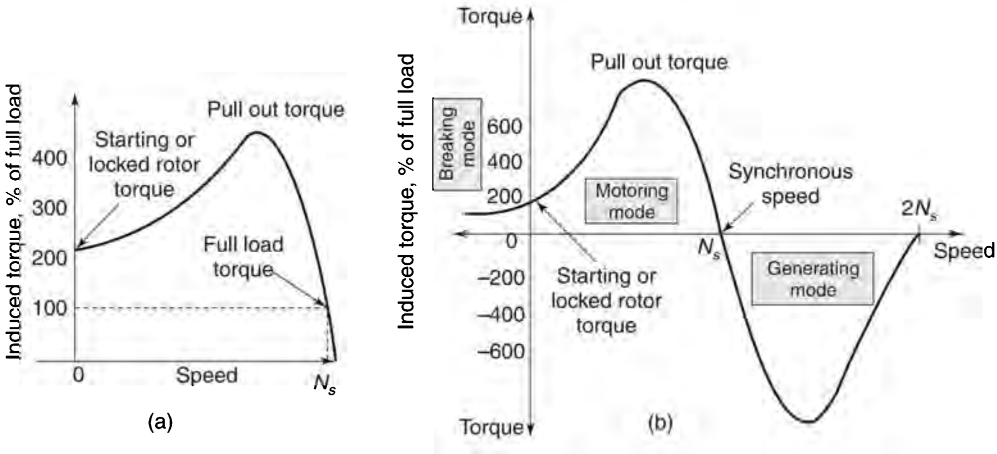


Fig. 19

From the two graphs of Fig. 19, some important information about the operation of the induction motor can be listed as below:

(i) The induced torque is zero at the synchronous speed.

(ii) The locked rotor torque may be different at different standstill position of the rotor with respect to the stator.

(iii) This curve is nearly linear between no load and full load. In this range, the value of the rotor resistance is much more than the rotor reactance. So, as the slip increases, the rotor current, the rotor magnetic field and the induced torque increase linearly.

(iv) There is a maximum torque called pull-out torque which is 2 to 3 times the full load torque which cannot be exceeded. If the load torque is equal to or more than breakdown torque, there is a rapid decrease in the developed torque by the machine and the speed suddenly decreases and the machine stops.

(v) The starting torque is slightly more than its full load torque, so this motor can start with connected load. If both torques are equal or starting torque (locked rotor torque) is less than the load torque, the motor does not start.

(vi) The torque on the motor for a given slip changes as the square of the applied voltage which is used to speed control of the induction motor.

(vii) If the motor is driven faster than the synchronous speed, the machine works as a generator.

(viii) If the motor is running in the direction opposite to the direction of the magnetic field, the induced torque in the machine stops the motor. This is called plugging.

The mechanical power is plotted as in Fig. .20. It is to be noted that the power supplied by the motor occurs at a speed different from the maximum torque.

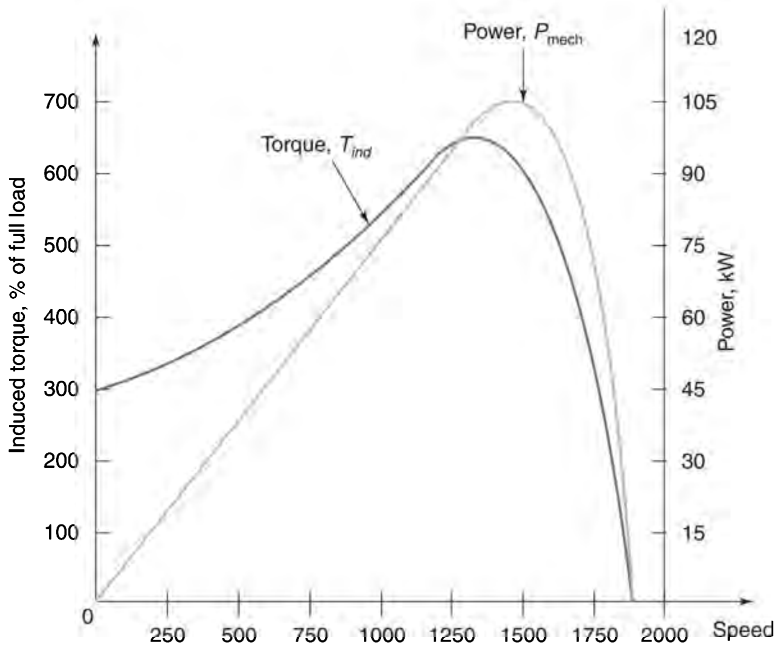


Fig. .20

The effect of changing rotor resistance on the torque speed curve of a wound rotor is shown in Fig. 21. From the curve, it is clear that as the resistance is increased, the pull-out speed of the motor decreases but the maximum torque always remains constant.

14 POWER FLOW OF THE INDUCTION MOTOR

We know that the power input of the induction motor is 3-phase AC supply. So, input power is given by

$$P_{IN} = \sqrt{3}V_L I_L \cos\phi = 3V_p I_p \cos\phi, \text{ where } \cos\phi \text{ is the input power factor.}$$

The various losses in the stator are:

(i) Copper losses, $P_{S\text{copper}} = 3I_1^2 R_1$

(ii) Core losses, $P_{S\text{core}}$

Power output of the stator, $P_{\text{OS}} = P_{\text{IN}} - P_{S\text{copper}} - P_{S\text{core}}$

This power is transferred to the rotor through air gap hence it is also known as air gap power P_{AG} of the motor.

So, output power of the stator = Air gap power = Rotor input power.

$$\therefore \quad P_{\text{OS}} = P_{AG} = P_{\text{INR}}$$

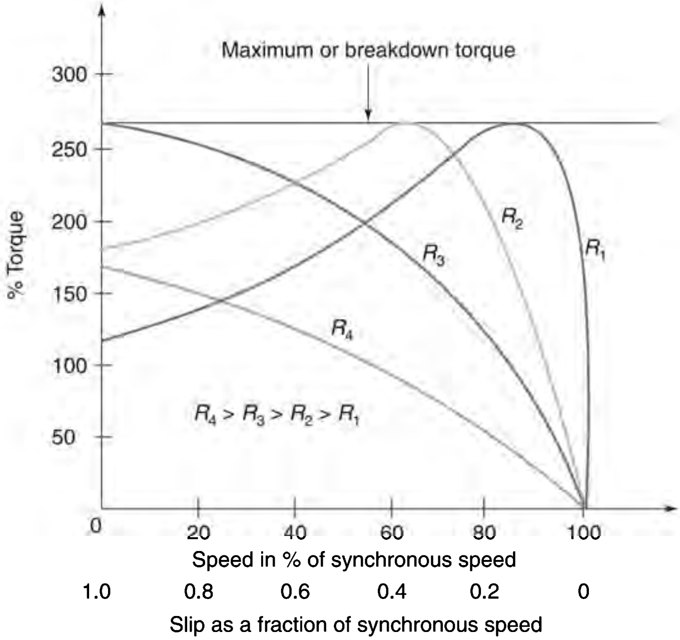


Fig. 21

The various losses in the rotor are:

(i) Copper losses, $P_{R\text{copper}} = 3I_2^2 R_2$

(ii) Core losses, $P_{R\text{core}}$

(iii) Friction and windage losses, P_{fw}

Stray load losses; P_{mis} consists of all losses which are not considered above, like the losses due to harmonics field.

Now, the mechanical power developed, P_{mech} can be obtained by subtracting the rotor copper losses from the rotor input power. The remaining power is changed from electrical to mechanical form which is called developed mechanical power P_{mech}.

$$\therefore \quad P_{\text{mech}} = P_{INR} - P_{R\text{copper}} = P_{AG} - P_{R\text{copper}}$$

$$\therefore \quad P_{\text{mech}} = P_{AG} - 3I_2^2 R_2$$

The output of the motor is given by $P_o = P_{\text{mech}} - P_{fw} - P_{\text{mis}}$

Now, during starting period, the rotor core losses are more but as the speed of the motor increases, these losses get reduced. A friction and windage loss at the starting is zero and as the motor gets speeded up, it increases. So, the sum of the friction and windage losses and the core losses is almost constant with any change in speed. Hence, these losses are sometimes bunched together and called as *rotational losses*.

$\therefore$ *Rotational losses*, $$P_{ROT} = P_{fw} + P_{R\text{core}} + P_{\text{misc}}$$

$\therefore$ *Output of motor*, $$P_o = P_{\text{mech}} - P_{ROT}.$$

Here P_o is the shaft power or useful power.

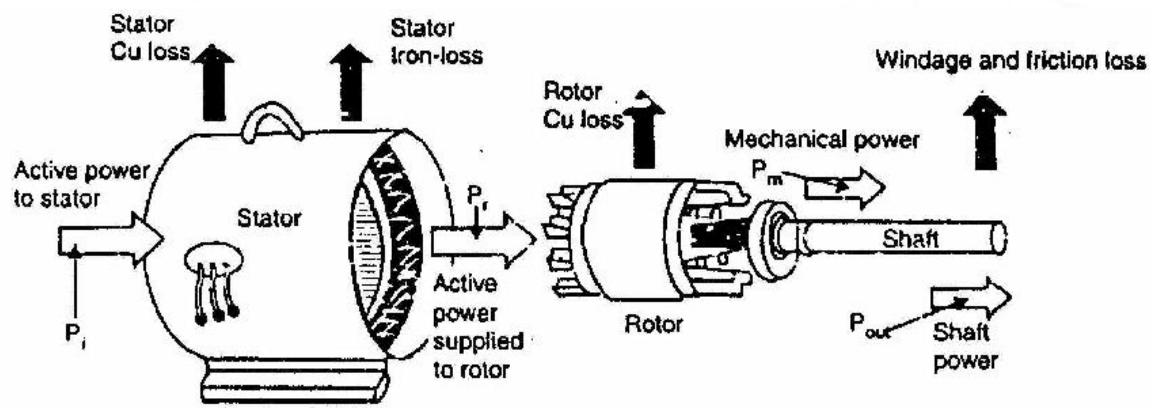


Fig 22 Power flow diagram of induction motor

Rotational loss is a purely mechanical quantity. Hence, it is not shown by any element in the equivalent circuit.

There is no method to show the core losses on the circuit model. Core losses of the motor consist of stator core losses and rotor core losses. The rotor core losses vary with rotor frequency and hence with the slip. Under normal conditions, the slip is 3% and hence rotor frequency is about 1.5 Hz. So, rotor core losses are negligible. Hence, all the core losses are bunched together and represented by resistor R_0 in the stator circuit.

15 INDUCTION MOTOR AS A TRANSFORMER

We know that an induction motor is a rotating transformer having an air gap and contains stator as a primary winding and short-circuited rotor as a secondary winding as shown in Fig. .23. The energy conversion takes place through induction.

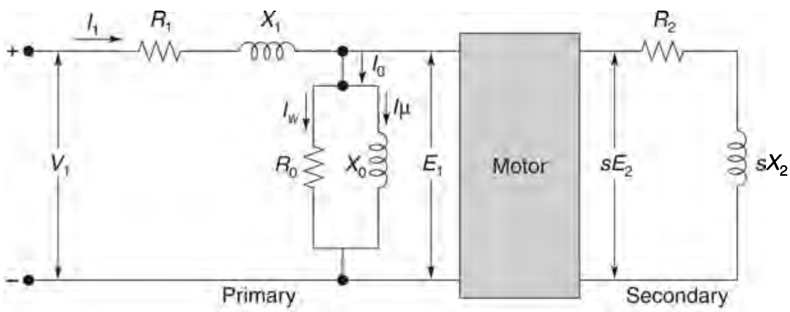


Fig. 23

The vector diagram is also same for the transformer. It is shown in Fig. 24.

$V_1 = E_1 + I_1(R_1 + jX_1)$ and $E_2 = I_2(R_2 + jsX_2)$. However, there is some important points of difference between the transformer and the induction motor, which are:

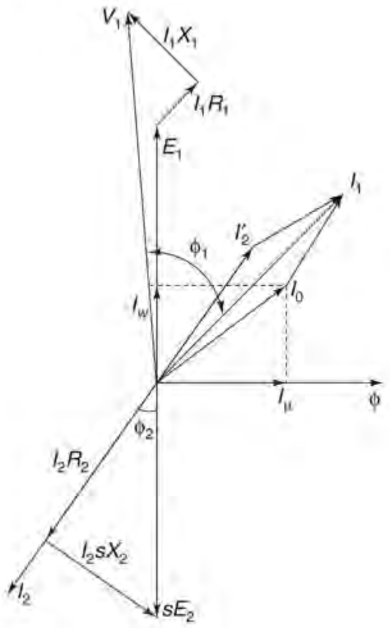


Fig. 24 Vector diagram of induction motor

(i) The magnetic leakage and leakage reactance of the stator and rotor of the induction motor are high as compared to the transformer.

(ii) Induction motor has an air gap hence the magnetizing current in the motor is higher than the transformer.

(iii) Because of the distributed winding in motor, the ratio of stator and rotor currents is not equal to the ratio of the turns per phase in the rotor and the stator.

(iv) Losses of the induction motor are more. Hence, the efficiency of the motor is lower than the transformer's.

15.1 Transformer and Induction Motor at No Load

Table 7.1 Comparison between a Transformer and an Induction Motor at No Load

Sl No.	*Transformer*	*Induction Motor*
01	Primary and secondary are placed on the stationary core.	Primary is on a stationary core called stator, and the secondary is on a rotating core called rotor.
02	The flux passes through a single core only.	The flux passes through stator, air gap and rotor.
03	Secondary winding is open or connected to the load.	Secondary is always shorted by a thick copper wire.
04	Draws less exciting current (2 – 6% of full load current).	Draws more exciting current (30 – 45% of full load current.)
05	Operates at high lagging power factor.	Operates at low lagging power factor.

Contd...

Contd...

06	Transformation ratio K = Secondary phase voltage/Primary phase voltage	K = Rotor phase voltage/Stator phase voltage
07	Secondary voltage is constant for all loads.	Rotor voltage is not constant but equal to sE2
08	Frequency of the primary and secondary are the same.	They are not same.
09	Load is electrical.	Load is mechanical.
10	No torque is produced.	Torque is produced.
11	No mechanical losses.	Mechanical losses are there.
12	High efficiency.	Low efficiency compared to the transformer.
13	Power is transmitted through the core.	Power is transmitted through the air gap.
14	Core losses are high.	Core losses are low as the frequency of the rotor voltage is low.
15	Core carries the same maximum flux under all load conditions.	Stator, air gap and rotor core carries flux.
16	Only one resultant flux in the core.	Only one resultant RMF in the air gap.
17	Secondary current changes as per the loading.	Rotor current changes as per the loading.
18	Secondary flux is counterbalanced by the primary by drawing extra current	Rotor flux is counterbalanced by the primary by drawing extra current
19	Load transfer takes place by counter flux.	Load transfer takes place by counter flux.
20	Leakage reactance of secondary varies with the load.	Rotor winding reactance varies with the slip.
21	It is a stationary machine; so there is no speed.	It is a rotating machine.

16 MEASURING THE SLIP

We know that a poly-phase induction motor always runs at a speed less than the synchronous speed. The difference between the actual speed of the motor and the synchronous speed is known as the slip. To measure the slip of the motor, generally following methods are used.

Generally, the tachometer (speedometer) is used to measure the speed of the motor directly. Tachometer may be analog or digital.

(i) By electro-mechanical counter

This is a very simple and direct method for measuring the slip. Here, a small synchronous motor is used that has the same number of poles as the induction motor. A cylinder having circular slip ring is fitted at the end of each shaft. A small contactor is connected to the slip ring and slip rings are connected with an electrical pulse counter as shown in Fig. ˙ 25.

Synchronous motor runs at the synchronous speed. So, the induction motor slips a revolution and the contactor closes the circuit and registers a pulse. The number of pulses per minute directly gives the number of slips in rpm.

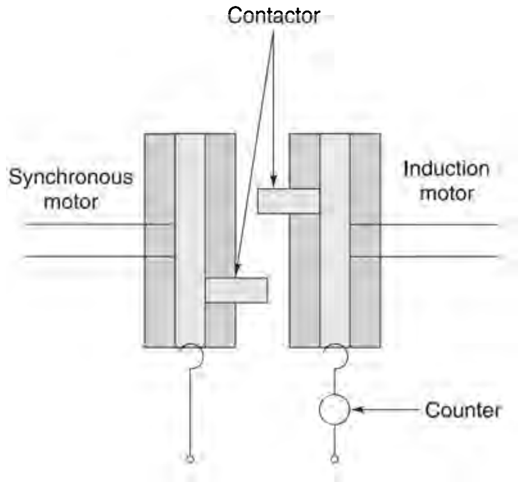


Fig. 25

(ii) By mechanical differential counter

In this method, a mechanical differential is used. The output gear of the differential rotates at a speed equal to the difference of two input gears which are connected to the synchronous motor and induction motor shaft. The main disadvantage of this system is that due to the gear arrangement, the induction motor is somewhat loaded.

(iii) By stroboscopic method

As shown in Fig. 26, this method uses a circular metallic disc. This disc is painted to produce black and white segments. The number of segments is equal to the number of poles in the induction motor. For a 4-pole motor, two white and two black segments are used. This disc is mounted on a shaft and illuminated by neon-filled stroboscopic lamp. The motor is started and the number of lines passing a fixed point is counted for one minute. If we divide this reading by the number of lines, it gives the slip speed, i.e., $N_s - N$. So we can find the slip from the equation, $s = \frac{N_s - N}{N_s} \times 100$.

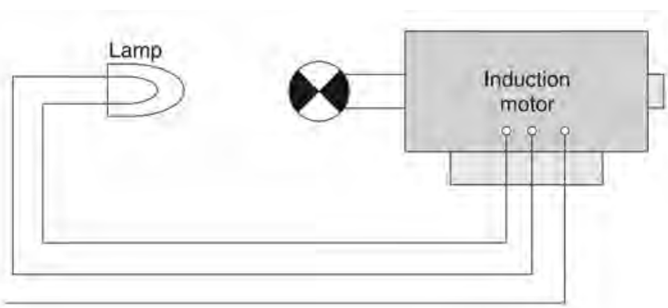


Fig. .26

Now-a-days, a stroboscope is used to measure the actual speed of the motor. In this instrument, a bulb flashes constantly, the flash speed of which is adjustable. When the number of flashes is equal to the speed of the motor, the motor looks stationary and the speed of the motor is known.

(iv) By comparing the rotor and stator frequencies

We know that $f_r = sf$, so the frequency is generally known or can be measured by connecting a frequency meter in the supply. The rotor frequency is so low that individual cycles can actually be counted. For this purpose, a DC moving coil milli-voltmeter with a central zero, is used. This voltmeter is connected between the two leads of the rotor (if the rotor is wound rotor) and the cycles are noted. Here, one should remember that one complete cycle equals the movement from zero to the maximum right, back to zero and on to a maximum left and then to zero again. Then, the slip can be calculated as $s = \frac{f_r}{f}$. However, in the squirrel cage rotor, it is not possible to connect the voltmeter.

17 DETERMINING THE EFFICIENCY

We know that the motor is a converter which converts electrical energy into mechanical energy for certain applications. In this process, the energy is lost as shown in Fig. 27.

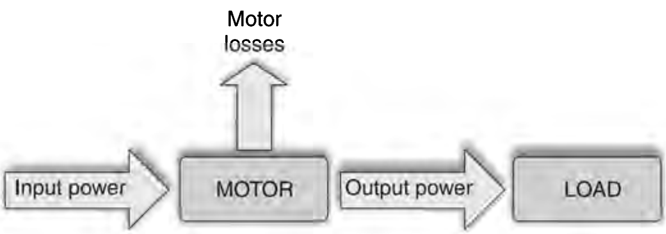


Fig. 27

The efficiency of a motor is greatly affected by its losses and improper designing and operating conditions. These losses can be minimized and the efficiency of the motor can be increased. Variations in losses can be in between 2 to 20%. Table 2 shows the types of losses for an induction motor.

The efficiency of a motor can be defined as "The ratio of a motor's useful output power to its total input power."

Thus,
$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}}$$

Table 2 Types of Losses in an Induction Motor

Types of Losses	*%age of Total Loss*
Fixed losses or core losses.	25
Variable losses—stator copper losses	34
Variable losses—rotor copper losses	21
Friction and windage losses	15
Stray load losses	5

Efficiency of small motors can be found directly by loading them. The input power and output power of the motor are measured by wattmeters and the efficiency is calculated. For large motors, this method is not suitable because it is very difficult to arrange a large load. Secondly, a large amount of power is wasted in direct loading. Hence, indirect methods are used to calculate the efficiency of the poly-phase induction motors. The following tests are performed. Circuit parameters of the equivalent circuit are also determined from these tests.

(1) No load test

The following parameters are determined from this test. The connection diagram is shown in Fig. 28.

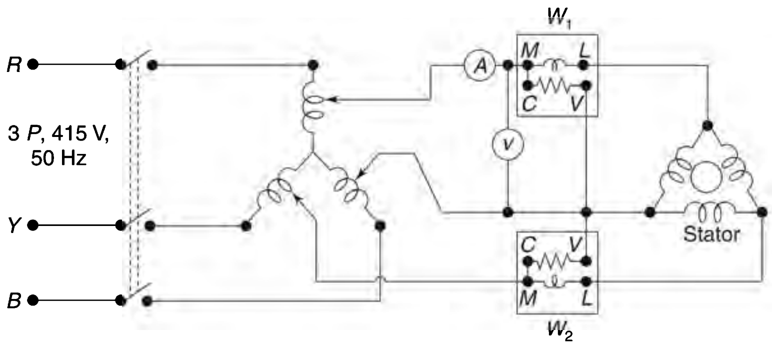


Fig. 28 No load test of induction motor

(a) No load current I_0.

(b) No load power factor, cos ϕ_0.

(c) Windage and friction losses.

(d) No load core losses.

(e) No load input power.

(f) No load resistance R_0 and reactance X_0

In this test, the motor is run at the rated voltage and frequency. The input power is measured by two wattmeters. An ampere-meter and voltmeter are also connected as in Fig. 28. When the machine is on no load, the slip is close to zero. So the circuit to the right of the shunt branch of the complete equivalent circuit is open circuit. Since the machine is running at no load, the total input power is equal to the total losses that occur in the motor. So, the total input power contains stator copper losses, stator core losses and friction, and windage losses. Stator core losses and friction, and windage losses are called constant or fixed losses because they are independent of the load.

Hence, total power input = $W_0 = W_1 + W_2 = P_{constant}$

Since the power factor of the motor under no load is very low, generally less than 0.5, one wattmeter will show negative reading. Therefore, the direction of the current coil is reversed.

Here, input power to the motor, $W_0 = \sqrt{3} V_0 I_0 \cos \phi_0$

V_0 is the line voltage, V_p is phase voltage $= \dfrac{V_0}{\sqrt{3}}$, and I_0 is the no load current

$$\cos \phi_0 = \frac{W_0}{\sqrt{3} V_0 I_0}$$

$$I_\mu = I_0 \sin \phi_0$$

$$I_\omega = I_0 \cos \phi_0 = I_0 \times \frac{W_0}{\sqrt{3} V_0 I_0} = \frac{W_0}{\sqrt{3} V_L}$$

$$R_0 = \frac{V_p}{I_\omega}$$

$$X_0 = \frac{V_p}{I_\mu}$$

Friction and windage losses of the motor can be separately found by taking a number of readings of W_0 at different stator applied voltages. Then a graph between W_0 and V_0 is plotted which is nearly parabolic at the normal voltage, since iron losses are directly proportional to the square of the applied voltage. The curve is extended to cut the vertical axis at A (Fig. .29). At the vertical axis, $V = 0$ and hence OA represents the voltage independent losses which are friction and windage losses.

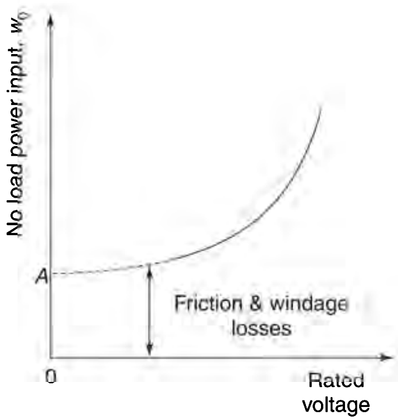


Fig. '29 Determination of friction and windage loss

(2) Blocked rotor test

This test is performed to find:

(a) Short circuit current, I_{sc}
(b) Power factor on short circuit, ϕ_{sc}
(c) Full load copper loss
(d) Total equivalent resistance and reactance of the motor referred to the stator.

In this test, the shaft of the motor is blocked and the rotor winding is short circuited if the motor is slip ring type. In the cage motor, the rotor is permanently short circuited. This test is also called *locked rotor test.*

In this test, reduced voltage is applied to the stator so that full load current of the motor flows in the stator. The power input here is equal to the copper losses of the rotor and the stator, because reduced voltage is applied and the rotor is blocked. Hence, core losses and mechanical losses are neglected.

Thus, Total power input on short circuit, W_{sc} = Total copper loss

I_{sc} = Line current on the short circuit.

V_{sc} = Line voltage on short circuit.

$$\therefore \quad W_{sc} = \sqrt{3} V_{sc} I_{sc} \cos \phi_{sc} = 3 I_{sc}^2 R_{eq}$$

where ϕ_{sc} is the power factor on the short circuit.

$$\therefore \quad R_{eq} = \frac{W_{sc}}{3 I_{sc}^2}$$

$$Z_{eq} = \frac{V_{\text{scp}}}{I_{\text{scp}}}$$

Equivalent reactance of the motor referred to the stator, $X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$

Now the stator resistance per phase is measured by using simple voltmeter-ammeter method. Due to skin effect, the multiplying factor of 1.25 may be used to get the AC resistance.

The efficiency of the motor by using the equivalent circuit can be obtained by the following procedure. For this purpose, three tests are required.

(a) Stator resistance measurement.
(b) No load test to get the constant loss.
(c) Load test to measure the total input and the slip.

After obtaining data from these three tests, we can calculate the efficiency as under:

(i) Measure the input power of the stator
(ii) Calculate the stator copper loss at no load.
(iii) Subtract the no load copper loss from the no load input power which is the constant loss containing the sum of friction and windage loss and core loss.
(iv) Load the motor and measure the input power, current and slip.
(v) Calculate the stator copper loss at the given load.
(vi) Subtract the copper loss from the motor input power to calculate rotor input power.
(vii) Multiply the rotor input power by slip to get the rotor copper loss.
(viii) Calculate the total copper loss in the rotor by adding rotor copper loss and constant loss.
(ix) Subtract the total rotor loss from the rotor input power to get the rotor output power.

However, the efficiency calculated from the equivalent circuit is more accurate. The efficiency calculated by no load and block rotor test gives lower value.

Example .19 A 3-phase star-connected 415 V, induction motor gives the following test results during the test:

No load test: 415 V, 20 A, W_1 = 5200 W, W_2 = – 3200 W.

Blocked rotor test: 60 V, 60 A, W_1 = 2200 W, W_2 = 700 W.

Find the circuit parameters if the resistance per phase is found to be 0.25 Ω.

Solution:

(i) $W_0 = W_1 - W_2 = 5200 - 3200 = 2000$ W

$$V_0 = \frac{415}{\sqrt{3}} = 240 \text{ V}$$

$$\cos \phi_0 = \frac{W_0}{3V_0 I_0} = \frac{2000}{3 \times 240 \times 20} = 0.13$$

$$R_0 = \frac{V_0}{I_0 \cos \phi_0} = \frac{240}{20 \times 0.02} = 50 \ \Omega$$

$$X_0 = \frac{V_0}{I_0 \sin \phi_0} = \frac{240}{20 \times 0.970} = 12.37 \ \Omega$$

(ii) $W_{sc} = 2200 + 700 = 2900$ W

$$V_{sc} = \frac{60}{\sqrt{3}} = 34.64 \text{ V}$$

$$\cos s_c = \frac{2900}{3 \times 34.64 \times 60} = 0.46$$

$$R_{eq} = \frac{W_{sc}}{3\, I_{sc}^2} = \frac{2900}{3 \times 60^2} = 0.26 \ \Omega$$

$$Z_{eq} = \frac{V_{sc}}{I_{sc}} = \frac{34.64}{60} = 0.577 \ \Omega$$

$$X_{eq} = \sqrt{0.577^2 - 0.26^2} = 0.515$$

Example .20 A 3-phase, 230 V, star-connected, 50 hp motor gives the following test results:

No load test: 230 V, 25 A, 1500 W

Blocked test: 75 V, 50 A, 3000 W

Find the circuit elements of the equivalent circuit of the induction motor if the stator resistance is 0.015 Ω per phase. Consider friction and windage loss to be 350 W.

Solution:

(i) *No load test*:

Stator copper loss at no load = $3I_0^2 R_1 = 3 \times 25^2 \times 0.015 = 28.12$ W

Core loss = Constant loss – No load copper loss – Mech. loss = 500 – 350 – 28.12 = 1122 W

∴ Core loss per phase = $\frac{1122}{3} = 374$ W

Phase voltage = $\frac{230}{\sqrt{3}} = 132.79$ V

$$\cos \phi_0 = \frac{1500}{\sqrt{3} \times 132.79 \times 25} = 0.26, \quad \phi_0 = 74.92 \quad \text{and} \quad \sin \phi_0 = 0.965$$

$$R_0 = \frac{132.79}{25 \times 0.26} = 20.42\ \Omega$$

$$X_0 = \frac{132.79}{25 \times 0.965} = 5.50\ \Omega$$

(ii) *Blocked rotor test*:

Volt per phase $= \frac{75}{\sqrt{3}} = 43.30$ V

$$\cos s_c \frac{2900}{\sqrt{3} \times 34.64 \times 60} = 0.80$$

$$R_{eq} = \frac{W_{sc}}{3I_{sc}^2} = \frac{3000}{3 \times 50^2} = 0.4\ \Omega$$

$$Z_{eq} = \frac{V_{sc}}{I_{sc}} = \frac{43.30}{50} = 0.87\ \Omega$$

$$X_{eq} = \sqrt{0.87^2 - 0.4^2} = 0.77\ \Omega$$

18 FACTORS AFFECTING THE EFFICIENCY OF THE MOTOR

(a) *Age of motor*: New motors are more efficient.

(b) *Capacity of motor*: Just like other equipment, motor efficiency is increased with the rated capacity.

(c) *Speed of motor*: Higher speed motors are usually more efficient.

(d) *Type of motor*: For example, efficiency of the squirrel cage motors is generally more than slip-ring motors.

(e) *Temperature*: It also affects the efficiency of the motor. For example, totally-enclosed fan-cooled (TEFC) motors gives more efficiency than that of screen protected drip-proof (SPDP) motors.

(f) *Rewinding of motor*: If the motor is rewound, its efficiency is decreased.

(g) *Load.*

When we assess the motor, load and efficiency must be determined. Generally, full load efficiency of the motor is given on the name plate by the manufacturer all over the world. However, when a motor is used for a long time, its efficiency cannot be determined because sometimes nameplates of motors are lost or painted over.

19 CIRCLE DIAGRAM

Performance of the induction motor can be easily understood by circle diagram under all operating conditions. The circle diagram of the induction motor is drawn from the data of approximate equivalent circuit as in Fig. 30.

The equivalent circuit is shown in Fig. 30 where the excitation branch is shifted left. This is justified when the stator impedance drop caused by no load current is neglected.

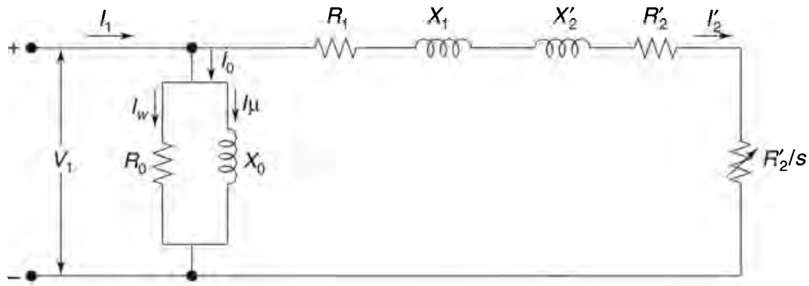


Fig. ..30 Approximate equivalent circuit of induction motor

The current I_2' can be written as

$$I_2' = \frac{V_1}{\sqrt{\left(R_1 + R_2' + \left(\frac{R_2'}{s}\right)\right)^2 + (X_1 + X_2')^2}}$$

It lags behind the applied voltage by an angle ϕ_2, such that

$$\sin \phi_2 = \frac{X_1 + X_2'}{\sqrt{\left(R_1 + R_2' + \left(\frac{R_2'}{s}\right)\right)^2 + (X_1 + X_2')^2}}$$

Combining the above two equations, we get

$$I_2' = \frac{V_1}{(X_1 + X_2')} \sin \phi_2$$

This is the equation of a circle with a diameter of $\frac{V_1}{(X_1 + X_2')}$. By changing the load, $\sin \phi_2$ will change which will cause change in rotor current I_2' as shown as under Fig. 31(a). The locus of input current I_1 can be found by adding a constant current I_0 and I_2' as in Fig. 31(b).

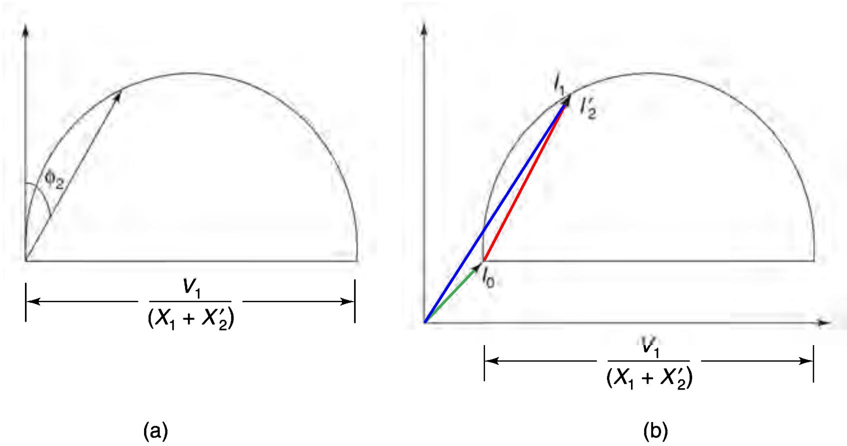


Fig. 31

19.1 Construction of the Circle Diagram

To construct the circle diagram of the motor, data from no load and blocked rotor tests are required. From those data, circle diagrams can be drawn as under.

(i) We get V_0, I_0, W_0 and cos ϕ_0 from no load test, and V_{sc}, I_{sc}, W_{sc} and cos ϕ_{sc} from blocked rotor test. I_{sc} is the short circuit current at reduced voltage V_{sc}. So, the short circuit current at normal voltage I_{scn} can be obtained by $I_{scn} = \frac{V}{V_{sc}} \times I_{sc}$, where V is the normal voltage.

(ii) The normal voltage applied to the stator V_1 in equivalent circuit is taken along the Y-axis. The no load current I_0 is drawn at angle ϕ_0 from OV$_1$. The no load current is shown by OO′ in Fig. .32. The voltage and current scales are chosen arbitrarily, but power scale and torque scales are derived as:

Power scale: watt per cm = V_1 × amperes per cm.

Torque scale: N.m per cm = $\frac{V_1}{\omega_s}$ × amperes per cm.

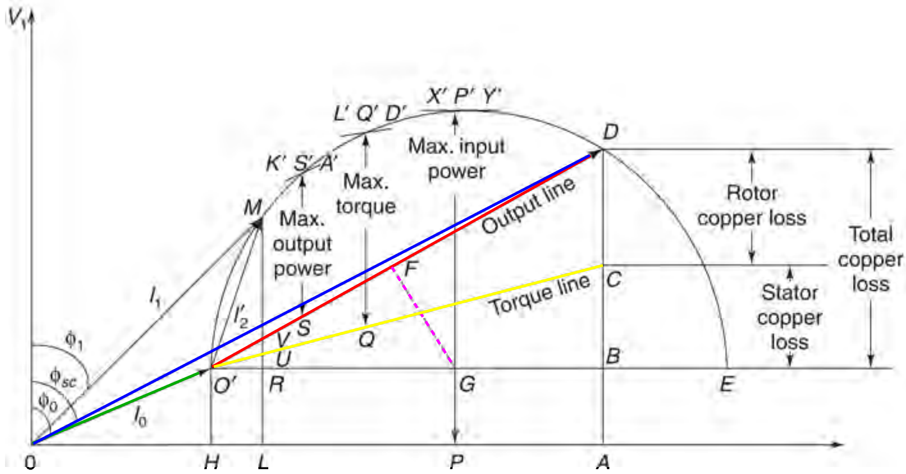


Fig. .32 Circle diagram of induction motor

(iii) Line OA is drawn perpendicular to the voltage phasor OV$_1$.

(iv) Phasor OD is drawn at an angle ϕ_{sc} from phasor OV$_1$ (lagging). So, phasor OD indicates the line current at the rated voltage when the motor is blocked.

(v) Points O′ and D are on the circle. To get the centre of this circle, line O′E is drawn parallel to the line OA. Now O′D is joined. FG is drawn perpendicular to O′D which bisects O′D at point F and intersects line O′E at point G. With G as centre and O′G as radius, draw a semicircle O′MP′DE.

(vi) Since the voltage phasor is drawn vertically, all the vertical distances give active components of currents and power to different scale, i.e., O′H gives active component of the no load current which supplies core losses, friction & windage losses and small amount of no load copper losses. O′H also gives the input power at no load to some other scale. Similarly, the vertical component DA of OD represents the active component of short circuit current I_{sc} which also represents the input power on short circuit to some other scale.

(vii) Hence, DA gives the input power when the rotor is blocked. This input power gives the core losses and stator–rotor copper losses. Line DB represents the stator and rotor copper losses when BA = O′H subtracted from DA.

(viii) Therafter, line DB is divided by C into two segments such that $\frac{\text{DC}}{\text{CB}} = \frac{R'_2}{R_1}$. In case of squirrel cage motor, stator resistance R_1 is measured and CB is made equal to stator copper loss ($3I_{sc}^2 R_1$). If motor is wound rotor type, line DB is directly divided into two parts such that $\frac{\text{DC}}{\text{CB}} = \frac{R'_2}{R_1}$.

(ix) After getting point C, it is joined with O′. O′C is a torque line.

19.2 Some Important Lines on the Circle Diagram

(i) In the circle diagram, line O′D represents the output line or mechanical power developed line.

(ii) If a line X′P′Y′ is drawn parallel to the line LPA and tangentially to the semicircle then the vertical intercept P' P between this tangential line X′P′Y′ represents the maximum power input to the motor.

(iii) If a line L′Q′D′ is drawn parallel to the torque line UQC and tangentially to the semicircle, then the vertical intercept Q′Q gives the maximum torque.

(iv) If a line K′S′A′ is drawn parallel to the output line O′D similarly, S′S gives the maximum output power.

(v) At any load current I_1 (represented by OM), the secondary current referred to the primary is I'_2 (represented by O′M) being equal to phasor difference of $(I_1 - I_0)$. LM represents the energy component of the load current I_1.

From the circle diagram,

No load current per phase = OO′

Input current per phase = OM

Rotor current per phase referred to stator = O′M

Motor input power watt per phase = LM

Constant losses watt per phase = LR

Stator copper loss per phase = RU

Rotor input power per phase = UM

Rotor copper loss = UV

Air gap power in watt per phase = UM on power scale and shaft torque in Nm on torque scale.

Shaft output in watt per phase = MV

Input power factor $= \frac{\text{ML}}{\text{OL}}$

Motor output per phase = MV

$$\text{Per unit efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{VM}}{\text{LM}}$$

$$\text{Per unit slip, } s = \frac{\text{Rotor copper loss}}{\text{Rotor input}} = \frac{\text{UV}}{\text{UM}}$$

Starting torque in Nm per phase = DC

$$\text{Power factor } \cos \phi_1 = \frac{\text{LM}}{\text{OM}}$$

Example .21 A 50 kW, 4-pole, 50 Hz, 415 V, 3-phase slip ring motor gives the following test results:

No load test: 415 V, 25 A, power factor = 0.15

Blocked rotor test: 175 V, 150 A, power factor = 0.3

The ratio of stator to rotor copper loss on the short circuit is 5 : 4. Draw the circle diagram and find (i) full load current, (ii) maximum torque and maximum input power, (iii) slip at full load, and (iv) efficiency.

Solution: Voltage = 415 V

No load current, $I_0 = 25$ A.

No load power factor, $\phi_0 = 0.15$

No load phase angle, $\phi_0 = \cos^{-1} 0.15 = 81.37°$

Short circuit voltage, $V_{sc} = 175$ V

Short circuit power factor, $\phi_{sc} = 0.3$, short circuit phase angle, $\phi_{sc} = 72.54°$

Short circuit current, $I_{sc} = 150$ A

$$\text{Short circuit current at normal voltage, } I_{\text{scn}} = \frac{V}{V_s} \times I_{sc} = \frac{415 \times 150}{175} = 335.71 \text{ A}$$

Short circuit power input with this current, $W_{sc} = \sqrt{3} \times VI_{\text{scn}} \cos \phi_{sc}$

$W_{sc} = \sqrt{3} \times 415 \times 335.71 \times 0.3 = 72392.61$ W

Let the current scale be taken as 15 A per cm. The circle diagram is shown in Fig. 33.

The procedure for drawing the circle diagram is as under:

(i) Voltage is considered as the reference vector on *Y*-axis. The no load current, shown by OO′, $I_0 = 25$ A is drawn at an angle of 81.37° with *Y*-axis. Hence, $I_0 = \frac{25}{15}$ = 1.66 cm.

(ii) Phasor OD is the I_{scn} and its value is $\frac{335.71}{15} = 22.38$ cm. It is drawn at an angle of 72.54° with *Y*-axis.

(iii) O′E is drawn parallel to the *X*-axis and FG is the bisector of the O′D.

(iv) Taking *G* as center and O′G as radius, draw the semicircle.

(v) DA gives the input power at short circuit with the normal voltage. It is measured and it gives 7.2 cm and it indicates 72392.61 W, hence, 1 cm = $\frac{72392.61}{7.2}$ = 10055 W.

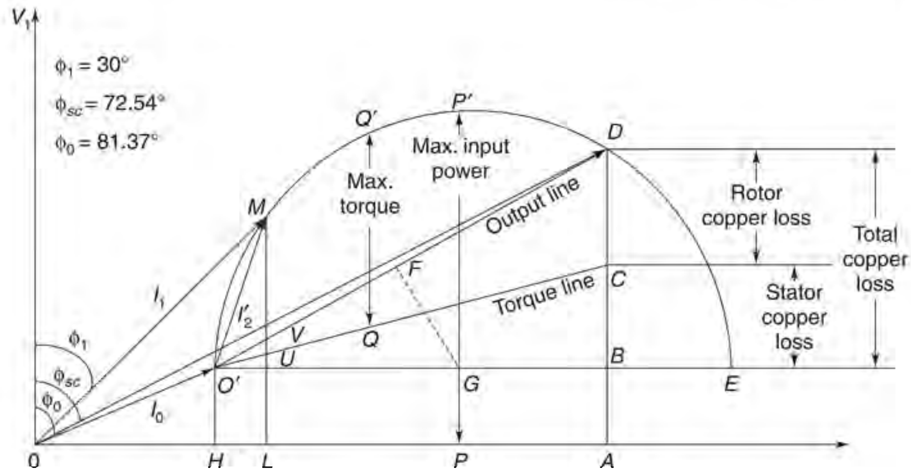


Fig. 33

(vi) Motor output is 50 kW. Hence, as per above power scale, the interception between semicircle and out line O′D is $\frac{50000}{10055} = 4.97$ cm.

So, the vertical line ML can be found which is equal to the 4.25 cm. Point M is the full load operating point.

(vii) Full load current is given by OM and its value is measured as 4.9 cm.

Hence, OM = 4.9 × 15 = 73.5 A.

Power factor ϕ is measured from circle diagram as $\phi = 30° = \cos 30 = 0.86$

(viii) The stator copper loss $= \frac{5}{4} = 1.25$ rotor copper loss.

Total copper loss = Stator copper loss + 1.25 rotor copper loss = 2.25 rotor copper loss.

(ix) From the circle diagram, total copper loss = DB = 7 cm, and rotor copper loss = DC

Hence, Total stator copper loss = BC, and thence, total copper loss, DB = 2.25 × DC

∴ $DC = \frac{DB}{2.25} = \frac{7}{2.25} = 3.11$ cm and CB = DB – DC = 7 – 3.11 = 3.88 cm.

The line O′C represents the torque line.

(x) To find the maximum torque, draw a line GQ′ which is perpendicular to the torque line and if it is drawn vertically downward from point Q′, it will intersect the torque line at point Q.

Line QQ′ indicates the maximum torque and its value is 9.5 cm.

∴ Maximum torque = QQ′ × Power scale = 10055 × 9.5 = 95522.5 syn watts.

(xi) To get maximum input power, line PP′ is drawn from the centre *G* of the semicircle.

∴ Maximum input power = PP′ × Power scale = 11.6 × 10055 = 116.638 kW

(xii) Slip at full load, $= \dfrac{\text{VU}}{\text{MU}} = \dfrac{0.1}{3.9} = 0.0256 = 2\%$

(xiii) Efficiency at full laod, $= \dfrac{\text{MV}}{\text{ML}} = \dfrac{3.8}{4.25} = 0.89$ pu $= 89\%$

Example .22 A 3-phase, 4-pole, 50 Hz, 500 V, induction motor has the following test data:

No load test: 500 V, 15 A, 1500 W.

Blocked rotor test: 250 V, 60 A, 7500 W.

The stator loss at standstill is 60% of the total copper loss of the motor and the full load current is of 35 A. Draw the circle diagram and determine (i) power factor, slip, output, efficiency speed and torque at full load, (ii) maximum power factor, (iii) starting torque, (iv) maximum power output, (v) maximum input power, (vi) maximum torque in synchronous watt and slip for maximum torque.

Solution: Applied voltage, $V_0 = 500$ V.

No load current, $I_0 = 15$ A

No load input, $W_0 = 1500$ W

No load power factor, $\phi_0 = \dfrac{1500}{\sqrt{3} \times 500 \times 15} = 0.115$

No load phase angle, $\phi_0 = 83.39°$

Short circuit voltage, $V_{sc} = 250$ V

Short circuit current, $I_{sc} = 60$ A

Short circuit input power, $W_{sc} = 7500$ W

Short circuit current with normal voltage, $I_{scn} = \dfrac{V}{V_{sc}} I_{sc} = \dfrac{500}{250} \times 60 = 120$ A

Short circuit input power with normaly voltage, $W_{scn} = \left(\dfrac{I_{scn}}{I_{sc}}\right)^2 \times W_{sc} = \left(\dfrac{120}{60}\right)^2 \times 7500$

$= 30000$ W

Short circuit power factor, $\cos \phi_{sc} = \dfrac{W_{scn}}{\sqrt{3} V I_{scn}} = \dfrac{30000}{\sqrt{3} \times 500 \times 120} = 0.28$

$\therefore \quad \phi_{sc} = \cos^{-1} 0.028 = 73.73°$

Let the current scale be taken as 5 A per cm. The circle diagram is shown in Fig. 34.

No load current phasor OO′ $= \dfrac{10}{5} = 2$ cm drawn with angle of 83.39° with V.

Phase OA represents 120 A. Hence, $\dfrac{120}{5} = 24$ cm with angle of 88.39 with V.

As earlier, draw line O′E parallel to X-axis, draw a semicircule and get AD which is the input power on short circuit with normal voltage. It is of 10 cm and it is of 30000 W.

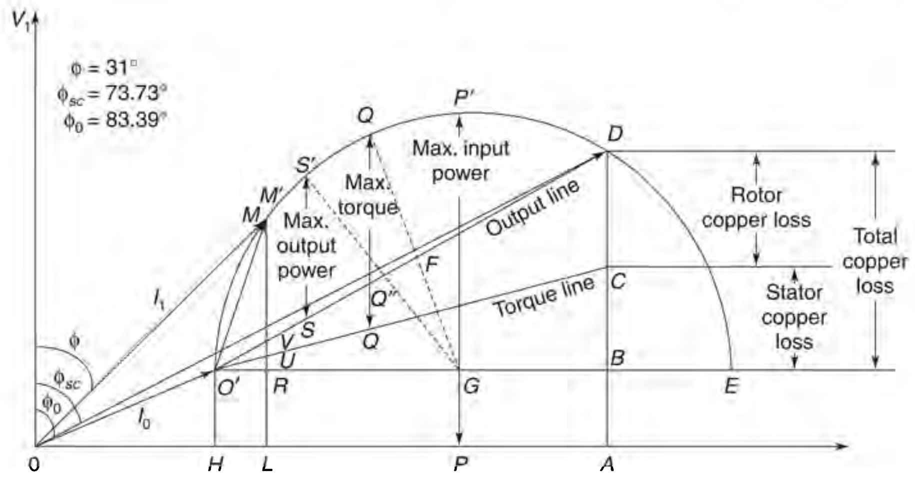


Fig. 34

Hence, Power scale = $\frac{30000}{10}$ = 3000 W

Full load current is 35 A; hence, $\frac{35}{5}$ = 7 cm. Taking 0′ as the centre with radius 7 cm, an arc is drawn which intersects the semicircle at point *M*. This represents full load condition. Join *M* and *O*.

Draw perpendicular ML from point *M* on the *X*-axis which cut the output line, torque line.

The stator copper loss is 60% of the total copper loss.

From the circle diagram,

Total copper loss = DB = 8 cm

∴ Stator copper loss = 0.6 × DB = 0.6 × 8 = 4.8 cm

∴ Rotor copper loss = 8 – 4.8 = 3.2 cm

At full load condition:

Power factor = $\frac{MR}{PO} = \frac{5.15}{6}$ = 0.8583 or power factor = cos ∠VOP = cos 31° = 0.8572

Slip = $\frac{UV}{MU} = \frac{0.15}{4.5}$ = 0.0333 = 3.33%

Output power, MV = 4.4 cm = 4.4 × 3000 W = 13200 W

Efficiency = $\frac{MV}{ML} = \frac{4.4}{5.15}$ = 0.85 pu = 85%

Speed at full load = $N_s(1 - s) = 120\frac{f}{p}(1 - s) = \frac{120 \times 50}{4}(1 - 0.033)$ = 1450 rpm

Torque at full load, = MU = 4.5 cm = 4.5 × 3000 = 13500 syn. W

Max. power factor is got by drawing a line from point O tangentially to the semicircle at point M′

∴ $\cos\phi_m = \angle VOM' = 31° = \cos 31° = 0.8572$

Starting torque, = DC = 3 cm = 3 × 3000 = 9000 syn. W

Max. power input can be obtained by drawing a line FG which cuts the semicircle at point S' and the line SS' is drawn vertically downward which cuts the horizontal line at point S. This line SS' is the

Maximum output which is of 7.5 cm = 7.5 × 3000 = 22500 syn. W

Max. power input can be get by drawing vertical line PP′ from point P and is value is of 11.2 cm

Max. power input = 11.2 × 3000 = 33600 syn. W

Max. torque is obtained by drawing GQ′ perpendicular to the O′C and dropping vertical line cutting at Q′ to the torque line. Line QQ′ is the max. torque and is of 8.5 cm.

Max. torque = 8.5 × 3000 = 25500 syn. W

Slip for maximum torque $= \dfrac{QQ''}{QQ'} = \dfrac{1.35}{8.5} = 0.15 = 15\%$

Example 7.23 A 3-phase, 415 V, induction motor gives the following test results:

No load test: 415 V, 10 A, 1300 W.

SC test: 160 V, 40 A, 4000 W.

If the normal rating is 15 kW, draw the circle diagram and find full load current, power factor and slip.

Solution: $\cos\phi_0 = \dfrac{1300}{\sqrt{3} \times 415 \times 10} = 0.18$

No load phase angle, $\phi_0 = \cos^{-1} 0.18 = 79.63°$

$$\cos\phi_{sc} = \frac{4000}{\sqrt{3} \times 160 \times 40} = 0.36$$

Short circuit phase angle, $\phi_{sc} = 68.89$

Short circuit current at normal voltage, $I_{scn} = \dfrac{V}{V_{sc}} \times I_{sc} = \dfrac{415}{160} \times 40 = 103.75$ A

Short circuit input power with normal voltage, $W_{scn} = \left(\dfrac{I_{scn}}{I_{sc}}\right)^2 \times W_{sc}$

∴ $$W_{scn} = \left(\frac{103.75}{40}\right)^2 \times 4000 = 26910 \text{ W}$$

Now taking current scale as 1 cm = 5 A

$$I_0 = \frac{10}{5} = 2 \text{ cm} \quad \text{and} \quad I_{scn} = \frac{103.75}{5} = 20.75 \text{ cm}$$

Both these currents are with their respective phase angle with V as shown in Fig. 7.35.

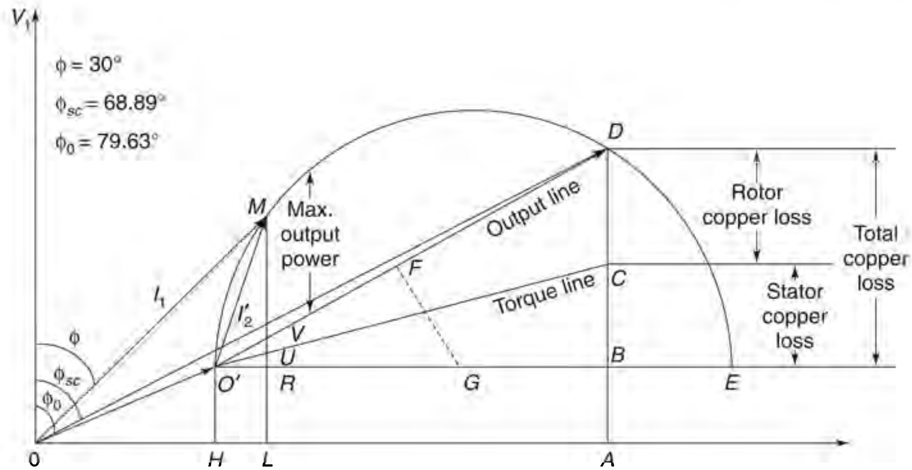


Fig. 7.35

AD which is input power on short-circuit with normal voltage, is of 10 cm and 26910 W.

Hence, power scale, $1 \text{ cm} = \frac{26910}{10} = 2691 \text{ W}$

Now output power is given as 15000 W and from circle diagram it is equal to MV

$$\text{Output power} = \frac{15000}{2691} = 5.57 \text{ cm}$$

Line current is given by OM and is measured as 6.5 cm = 6.5 × 5 = 32.5 A

Power factor is measured from circle diagram as 30°, cos 30° = 0.866

Slip is given by $\frac{UV}{UM} = \frac{0.25}{4.5} = 0.05$ pu = 5%