Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

ELECTROMAGNETICS I (EE305)

Electromagnetics (EM) - the study of electric and magnetic phenomena.

A knowledge of the fundamental behavior of *electric* and *magnetic* fields is necessary to understand the operation of such devices as resistors, capacitors, inductors, diodes, transistors, transformers, motors, relays, transmission lines, antennas, waveguides, optical fibers and lasers.

All electromagnetic phenomena are governed by a set of equations known as *Maxwell's equations*.

Maxwell's Equations

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial \boldsymbol{t}}$$

$$\nabla \cdot \boldsymbol{D} = \rho_v$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$\boldsymbol{E}$ - electric field intensity
$\boldsymbol{H}$ - magnetic field intensity
$\boldsymbol{D}$ - electric flux density
$\boldsymbol{B}$ - magnetic flux density
$\boldsymbol{J}$ - current density
ρ_v - volume charge density

Electromagnetics I

Static fields and applications, Introduction to Maxwell's equations.

Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

Vector Algebra

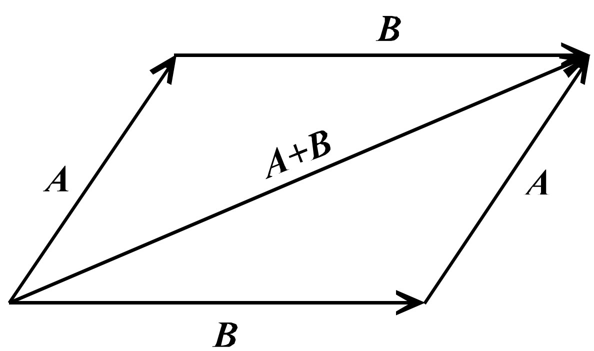
The quantities of interest appearing in Maxwell's equations along with other quantities encountered in the study of EM can almost always be classified as either a *scalar* or a *vector* (*tensors* are sometimes encountered in EM but will not be covered in this class).

Scalar - a quantity defined by magnitude only.
[examples: distance (x), voltage (V), charge density (ρ_v), etc.]

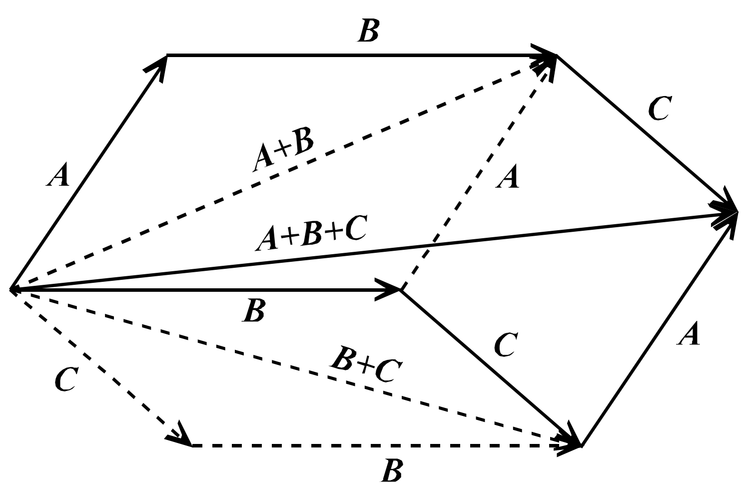
Vector - a quantity defined by magnitude and direction.
[examples: velocity ($\mathbf{v}$), current ($\mathbf{I}$), electric field ($\mathbf{E}$), etc.]

Note that vectors are denoted by boldface. The magnitude of a vector may be a real-valued scalar or a complex-valued scalar (phasor).

Vector Addition (Parallelogram Law)



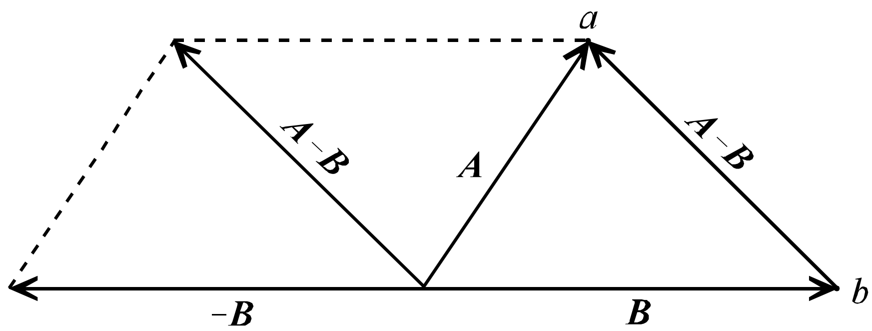
Commutative Law
$\mathbf{A}+\mathbf{B} = \mathbf{B}+\mathbf{A}$



Associative Law
$(\mathbf{A}+\mathbf{B})+\mathbf{C} = \mathbf{A}+(\mathbf{B}+\mathbf{C})$

Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

Vector Subtraction



$$\boldsymbol{A} - \boldsymbol{B} = \boldsymbol{A} + (-\boldsymbol{B})$$

Note:

(1) The magnitude of the vector $\boldsymbol{A}-\boldsymbol{B}$ is the separation distance d between the points a and b located by the vectors $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively $[d = |\boldsymbol{A}-\boldsymbol{B}| = |\boldsymbol{B}-\boldsymbol{A}|]$.

(2) The vector $\boldsymbol{A}-\boldsymbol{B}$ is the vector pointing from b (origination point) to a (termination point).

Multiplication and Division By a Scalar

$$a(\boldsymbol{A}+\boldsymbol{B}) = a\boldsymbol{A} + a\boldsymbol{B} \qquad \text{(Distributive law)}$$

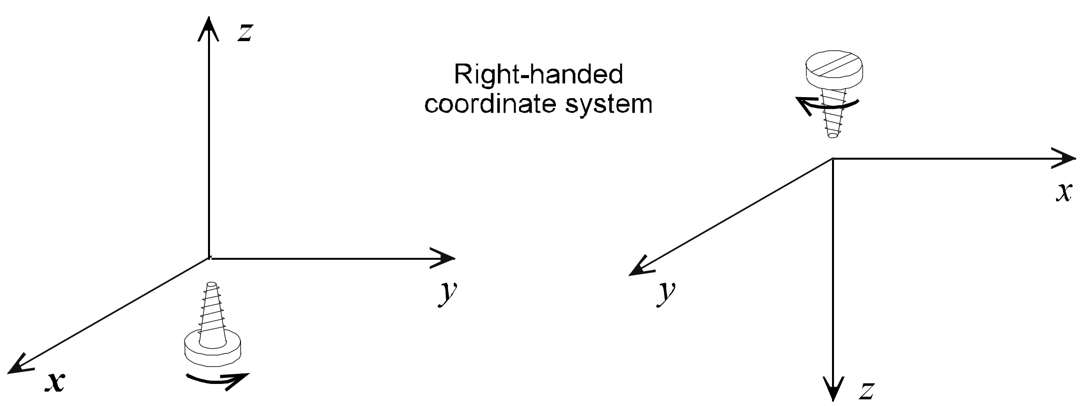
$$\frac{\boldsymbol{A}+\boldsymbol{B}}{a} = \frac{1}{a}\boldsymbol{A} + \frac{1}{a}\boldsymbol{B}$$

Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

Coordinate Systems

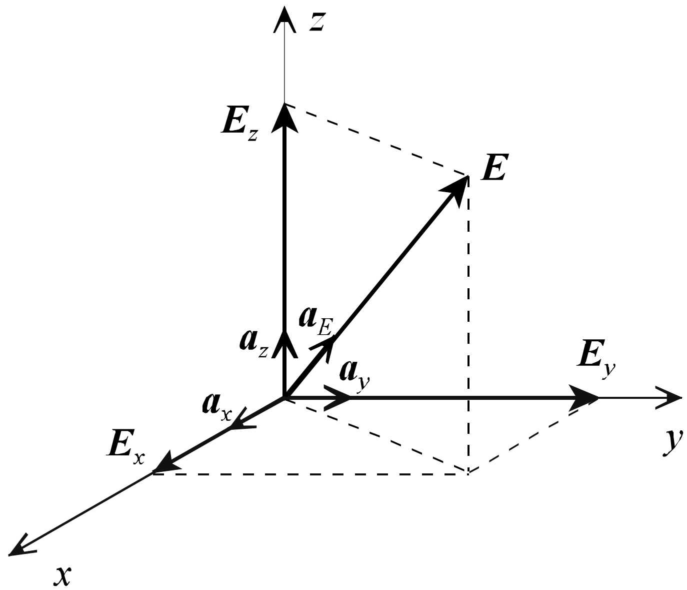
A coordinate system defines points of reference from which specific vector directions may be defined. Depending on the geometry of the application, one coordinate system may lead to more efficient vector definitions than others. The three most commonly used coordinate systems used in the study of electromagnetics are *rectangular* coordinates (or *cartesian* coordinates), *cylindrical* coordinates, and *spherical* coordinates.

Rectangular Coordinates



The rectangular coordinate system is an *orthogonal* coordinate system with coordinate axes defined by x, y, and z. The coordinate axes in an orthogonal coordinate system are mutually perpendicular. By convention, we choose to define rectangular coordinates as a *right-handed* coordinate system. This convention ensures that the three coordinate axes are always drawn with the same orientation no matter how the coordinate system may be rotated. If we position a right-handed screw normal to the plane containing the x and y axes, and rotate the screw in the direction of the x axis rotated toward the y axis, the direction that the screw advances defines the direction of the z axis in a right-handed coordinate system.

Component Scalars and Component Vectors



Given an arbitrary vector $\boldsymbol{E}$ in rectangular coordinates, the vector $\boldsymbol{E}$ can be described (using vector addition) as the sum of three *component vectors* that lie along the coordinate axes.

$$\boldsymbol{E} = \boldsymbol{E}_x + \boldsymbol{E}_y + \boldsymbol{E}_z$$

Component Vectors

The component vectors can be further simplified by defining *unit vectors* along the coordinate axes: $\boldsymbol{a}_x$, $\boldsymbol{a}_y$, and $\boldsymbol{a}_z$. These unit vectors have magnitudes of unity and directions parallel to the respective coordinate axis. The component vectors can be written in terms of the unit vectors as

$$\boldsymbol{E}_x = |\boldsymbol{E}_x|\,\boldsymbol{a}_x = E_x\boldsymbol{a}_x$$

$$\boldsymbol{E}_y = |\boldsymbol{E}_y|\,\boldsymbol{a}_y = E_y\boldsymbol{a}_y$$

$$\boldsymbol{E}_z = |\boldsymbol{E}_z|\,\boldsymbol{a}_z = E_z\boldsymbol{a}_z$$

$$\boldsymbol{E} = \boldsymbol{E}_x + \boldsymbol{E}_y + \boldsymbol{E}_z = E_x\boldsymbol{a}_x + E_y\boldsymbol{a}_y + E_z\boldsymbol{a}_z$$

Component Scalars

Thus, using *component scalars*, any rectangular coordinate vector can be uniquely defined using three scalar quantities that represent the magnitudes of the respective component vectors.

To define a unit vector in the direction of $\boldsymbol{E}$, we simply divide the vector by its magnitude.

$$\boldsymbol{a}_E = \frac{\boldsymbol{E}}{|\boldsymbol{E}|} = \frac{E_x\boldsymbol{a}_x + E_y\boldsymbol{a}_y + E_z\boldsymbol{a}_z}{\sqrt{E_x^2 + E_y^2 + E_z^2}} \qquad \left(\begin{array}{c}\text{unit vector in the}\\ \text{direction of } \boldsymbol{E}\end{array}\right)$$

where the magnitude of $\boldsymbol{E}$ is the diagonal of the rectangular volume formed by the three component scalars.

<u>Example</u> (Unit vector)

Given $\boldsymbol{E} = (x+y)\boldsymbol{a}_x + 3\boldsymbol{a}_y + z^2\boldsymbol{a}_z$, determine the unit vector in the direction of $\boldsymbol{E}$ at the rectangular coordinate location of (1,1,1).

$$E_x = x + y$$

$$E_y = 3$$

$$E_z = z^2$$

Note that the component scalars are functions of position (the direction of the vector changes with position).

$$\boldsymbol{a}_E = \frac{\boldsymbol{E}}{|\boldsymbol{E}|} = \frac{(x+y)\boldsymbol{a}_x + 3\boldsymbol{a}_y + z^2\boldsymbol{a}_z}{\sqrt{(x+y)^2 + 9 + z^4}} \qquad \left(\begin{array}{c}\text{unit vector as a}\\ \text{function of position}\end{array}\right)$$

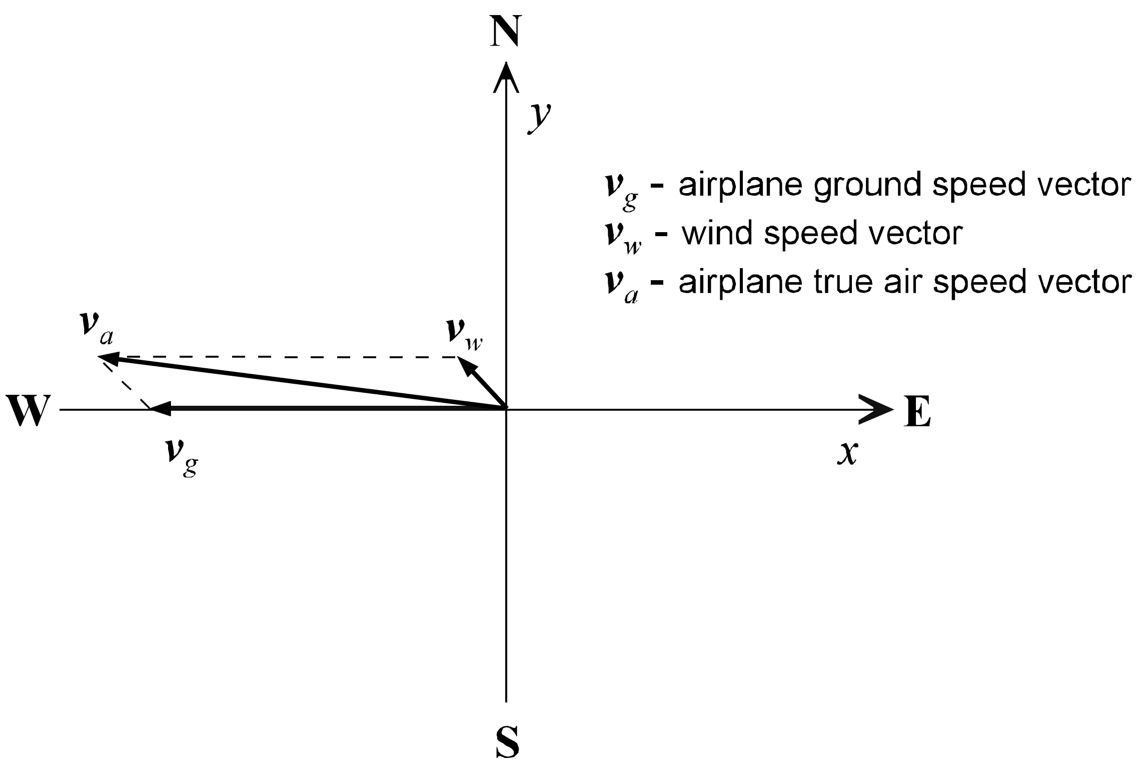
At the point (1,1,1) [$x = 1$, $y = 1$, $z = 1$],

$$\boxed{\boldsymbol{a}_E = \frac{1}{\sqrt{14}}(2\boldsymbol{a}_x + 3\boldsymbol{a}_y + \boldsymbol{a}_z)}$$

Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

Example (Vector addition)

An airplane with a ground speed of 350 km/hr heading due west flies in a wind blowing to the northwest at 40 km/hr. Determine the true air speed and heading of the airplane.



$$\boldsymbol{v}_g = 350(-\boldsymbol{a}_x) = -350\,\boldsymbol{a}_x$$

$$\boldsymbol{v}_w = 40\cos 45^o(-\boldsymbol{a}_x) + 40\sin 45^o(\boldsymbol{a}_y) = -28.3\,\boldsymbol{a}_x + 28.3\,\boldsymbol{a}_y$$

$$\boldsymbol{v}_a = \boldsymbol{v}_g + \boldsymbol{v}_w = -350\,\boldsymbol{a}_x - 28.3\,\boldsymbol{a}_x + 28.3\,\boldsymbol{a}_y = -378.3\,\boldsymbol{a}_x + 28.3\,\boldsymbol{a}_y$$

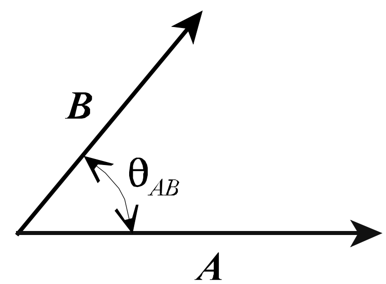
$$|\boldsymbol{v}_a| = \sqrt{(378.3)^2 + (28.3)^2} = 379.4 \text{ km/hr}$$

$$\theta = \tan^{-1}\frac{28.3}{378.3} = 4.28^o \text{ north of west}$$

Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

Dot Product
(Scalar Product)

The *dot product* of two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ (denoted by $\boldsymbol{A} \cdot \boldsymbol{B}$) is defined as the product of the vector magnitudes and the cosine of the smaller angle between them.



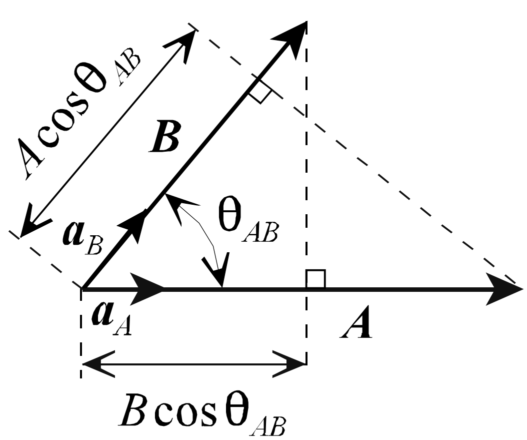
$$\boldsymbol{A} \cdot \boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}| \cos\theta_{AB} = AB\cos\theta_{AB}$$

$$\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{B} \cdot \boldsymbol{A} \qquad \text{(commutative law)}$$

The dot product is commonly used to determine the component of a vector in a particular direction. The dot product of a vector with a unit vector yields the component of the vector in the direction of the unit vector. Given two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ with corresponding unit vectors $\boldsymbol{a}_A$ and $\boldsymbol{a}_B$, the component of $\boldsymbol{A}$ in the direction of $\boldsymbol{B}$ (the *projection* of $\boldsymbol{A}$ onto $\boldsymbol{B}$) is found evaluating the dot product of $\boldsymbol{A}$ with $\boldsymbol{a}_B$. Similarly, the component of $\boldsymbol{B}$ in the direction of $\boldsymbol{A}$ (the *projection* of $\boldsymbol{B}$ onto $\boldsymbol{A}$) is found evaluating the dot product of $\boldsymbol{B}$ with $\boldsymbol{a}_A$.

$$\boldsymbol{A} \cdot \boldsymbol{a}_B = |\boldsymbol{A}|\,|\boldsymbol{a}_B| \cos\theta_{AB} = A\cos\theta_{AB}$$

$$\boldsymbol{B} \cdot \boldsymbol{a}_A = |\boldsymbol{B}|\,|\boldsymbol{a}_A| \cos\theta_{AB} = B\cos\theta_{AB}$$



The dot product can be expressed independent of angles through the use of component vectors in an orthogonal coordinate system.

$$\boldsymbol{A} = A_x\boldsymbol{a}_x + A_y\boldsymbol{a}_y + A_z\boldsymbol{a}_z$$

$$\boldsymbol{B} = B_x\boldsymbol{a}_x + B_y\boldsymbol{a}_y + B_z\boldsymbol{a}_z$$

$$\boldsymbol{A}\cdot\boldsymbol{B} = (A_x\boldsymbol{a}_x + A_y\boldsymbol{a}_y + A_z\boldsymbol{a}_z)\cdot(B_x\boldsymbol{a}_x + B_y\boldsymbol{a}_y + B_z\boldsymbol{a}_z)$$

$$= A_xB_x\,\boldsymbol{a}_x\cdot\boldsymbol{a}_x + A_xB_y\,\boldsymbol{a}_x\cdot\boldsymbol{a}_y + A_xB_z\,\boldsymbol{a}_x\cdot\boldsymbol{a}_z$$

$$+ A_yB_x\,\boldsymbol{a}_y\cdot\boldsymbol{a}_x + A_yB_y\,\boldsymbol{a}_y\cdot\boldsymbol{a}_y + A_yB_z\,\boldsymbol{a}_y\cdot\boldsymbol{a}_z$$

$$+ A_zB_x\,\boldsymbol{a}_z\cdot\boldsymbol{a}_x + A_zB_y\,\boldsymbol{a}_z\cdot\boldsymbol{a}_y + A_zB_z\,\boldsymbol{a}_z\cdot\boldsymbol{a}_z$$

The dot product of like unit vectors yields one ($\theta_{AB} = 0^\circ$) while the dot product of unlike unit vectors ($\theta_{AB} = 90^\circ$) yields zero. The dot product results are

$$\boldsymbol{a}_x\cdot\boldsymbol{a}_x = 1 \qquad \boldsymbol{a}_x\cdot\boldsymbol{a}_y = 0 \qquad \boldsymbol{a}_x\cdot\boldsymbol{a}_z = 0$$

$$\boldsymbol{a}_y\cdot\boldsymbol{a}_x = 0 \qquad \boldsymbol{a}_y\cdot\boldsymbol{a}_y = 1 \qquad \boldsymbol{a}_y\cdot\boldsymbol{a}_z = 0$$

$$\boldsymbol{a}_z\cdot\boldsymbol{a}_x = 0 \qquad \boldsymbol{a}_z\cdot\boldsymbol{a}_y = 0 \qquad \boldsymbol{a}_z\cdot\boldsymbol{a}_z = 1$$

The resulting dot product expression is

$$\boxed{\boldsymbol{A}\cdot\boldsymbol{B} = A_xB_x + A_yB_y + A_zB_z}$$

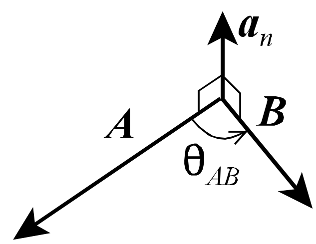
Dr. Ahmed Abdolkhalig
The University of Tobruk
Department of Electrical Engineering
www.ahmed.ucoz.org

Cross Product
(Vector Product)

The *cross product* of two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ (denoted by $\boldsymbol{A}\times\boldsymbol{B}$) is defined as the product of the vector magnitudes and the sine of the smaller angle between them with a vector direction defined by the *right hand rule*.

$$\boldsymbol{A}\times\boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}|\sin\theta_{AB}\,\boldsymbol{a}_n = AB\sin\theta_{AB}\,\boldsymbol{a}_n$$

$$\boldsymbol{A}\times\boldsymbol{B} = -\boldsymbol{B}\times\boldsymbol{A} \qquad \text{(not commutative)}$$



Note: (1) the unit vector $\boldsymbol{a}_n$ is normal to the plane in which $\boldsymbol{A}$ and $\boldsymbol{B}$ lie.
(2) $AB\sin\theta_{AB}$ = area of the parallelogram formed by the vectors $\boldsymbol{A}$ and $\boldsymbol{B}$.

Using component vectors, the cross product of $\boldsymbol{A}$ and $\boldsymbol{B}$ may be written as

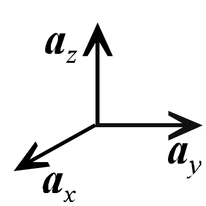
$$\boldsymbol{A} = A_x\boldsymbol{a}_x + A_y\boldsymbol{a}_y + A_z\boldsymbol{a}_z$$

$$\boldsymbol{B} = B_x\boldsymbol{a}_x + B_y\boldsymbol{a}_y + B_z\boldsymbol{a}_z$$

$$\begin{aligned}\boldsymbol{A}\times\boldsymbol{B} &= (A_x\boldsymbol{a}_x + A_y\boldsymbol{a}_y + A_z\boldsymbol{a}_z)\times(B_x\boldsymbol{a}_x + B_y\boldsymbol{a}_y + B_z\boldsymbol{a}_z)\\ &= A_xB_x\,\boldsymbol{a}_x\times\boldsymbol{a}_x + A_xB_y\,\boldsymbol{a}_x\times\boldsymbol{a}_y + A_xB_z\,\boldsymbol{a}_x\times\boldsymbol{a}_z\\ &+ A_yB_x\,\boldsymbol{a}_y\times\boldsymbol{a}_x + A_yB_y\,\boldsymbol{a}_y\times\boldsymbol{a}_y + A_yB_z\,\boldsymbol{a}_y\times\boldsymbol{a}_z\\ &+ A_zB_x\,\boldsymbol{a}_z\times\boldsymbol{a}_x + A_zB_y\,\boldsymbol{a}_z\times\boldsymbol{a}_y + A_zB_z\,\boldsymbol{a}_z\times\boldsymbol{a}_z\end{aligned}$$

The cross product of like unit vectors yields zero ($\theta_{AB} = 0^o$) while the cross product of unlike unit vectors ($\theta_{AB} = 90^o$) yields another unit vector which is determined according to the right hand rule. The cross products results are

$$\begin{array}{lll}\boldsymbol{a}_x\times\boldsymbol{a}_x = 0 & \boldsymbol{a}_x\times\boldsymbol{a}_y = \boldsymbol{a}_z & \boldsymbol{a}_x\times\boldsymbol{a}_z = -\boldsymbol{a}_y\\ \boldsymbol{a}_y\times\boldsymbol{a}_x = -\boldsymbol{a}_z & \boldsymbol{a}_y\times\boldsymbol{a}_y = 0 & \boldsymbol{a}_y\times\boldsymbol{a}_z = \boldsymbol{a}_x\\ \boldsymbol{a}_z\times\boldsymbol{a}_x = \boldsymbol{a}_y & \boldsymbol{a}_z\times\boldsymbol{a}_y = -\boldsymbol{a}_x & \boldsymbol{a}_z\times\boldsymbol{a}_z = 0\end{array}$$



The resulting cross product expression is

$$\boxed{\boldsymbol{A} \times \boldsymbol{B} = (A_y B_z - A_z B_y)\,\boldsymbol{a}_x + (A_z B_x - A_x B_z)\,\boldsymbol{a}_y + (A_x B_y - A_y B_x)\,\boldsymbol{a}_z}$$

This cross product result can also be written compactly in the form of a determinant as

$$\boxed{\boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}}$$

<u>Example</u> (Dot product / Cross product)

Given $\boldsymbol{E} = 3\boldsymbol{a}_y + 4\boldsymbol{a}_z$ and $\boldsymbol{F} = 4\boldsymbol{a}_x - 10\boldsymbol{a}_y + 5\boldsymbol{a}_z$, determine

(a.) the vector component of $\boldsymbol{E}$ in the direction of $\boldsymbol{F}$.

(b.) a unit vector perpendicular to both $\boldsymbol{E}$ and $\boldsymbol{F}$.

(a.) To find the vector component of $\boldsymbol{E}$ in the direction of $\boldsymbol{F}$, we must dot the vector $\boldsymbol{E}$ with the unit vector in the direction of $\boldsymbol{F}$.

$$\boldsymbol{a}_F = \frac{\boldsymbol{F}}{|\boldsymbol{F}|} = \frac{4\boldsymbol{a}_x - 10\boldsymbol{a}_y + 5\boldsymbol{a}_z}{\sqrt{4^2 + 10^2 + 5^2}} = \frac{1}{\sqrt{141}}(4\boldsymbol{a}_x - 10\boldsymbol{a}_y + 5\boldsymbol{a}_z)$$

The dot product of $\boldsymbol{E}$ and $\boldsymbol{a}_F$ is

$$\boldsymbol{E} \cdot \boldsymbol{a}_F = (3\boldsymbol{a}_y + 4\boldsymbol{a}_z) \cdot \frac{1}{\sqrt{141}}(4\boldsymbol{a}_x - 10\boldsymbol{a}_y + 5\boldsymbol{a}_z)$$

$$= \frac{1}{\sqrt{141}}[(3)(-10) + (4)(5)] = -\frac{10}{\sqrt{141}}$$

(Scalar component of $\boldsymbol{E}$ along $\boldsymbol{F}$)

The vector component of $\boldsymbol{E}$ along $\boldsymbol{F}$ is

$$(\boldsymbol{E}\cdot\boldsymbol{a}_F)\boldsymbol{a}_F = -\frac{10}{141}(4\boldsymbol{a}_x - 10\boldsymbol{a}_y + 5\boldsymbol{a}_z)$$

(b.) To find a unit vector normal to both $\boldsymbol{E}$ and $\boldsymbol{F}$, we use the cross product. The result of the cross product is a vector which is normal to both $\boldsymbol{E}$ and $\boldsymbol{F}$.

$$\boldsymbol{E}\times\boldsymbol{F} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ 0 & 3 & 4 \\ 4 & -10 & 5 \end{vmatrix} = (55\boldsymbol{a}_x + 16\boldsymbol{a}_y - 12\boldsymbol{a}_z)$$

We then divide this vector by its magnitude to find the unit vector.

$$\boldsymbol{a}_n = \frac{\boldsymbol{E}\times\boldsymbol{F}}{|\boldsymbol{E}\times\boldsymbol{F}|} = \frac{55\boldsymbol{a}_x + 16\boldsymbol{a}_y - 12\boldsymbol{a}_z}{\sqrt{55^2+16^2+12^2}} = \frac{1}{\sqrt{3425}}(55\boldsymbol{a}_x + 16\boldsymbol{a}_y - 12\boldsymbol{a}_z)$$

The negative of this unit vector is also normal to both $\boldsymbol{E}$ and $\boldsymbol{F}$.